DAMAGE EVALUATION IN BEAM STRUCTURES USING VIBRATION DATA

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الملخص

تم في هذه الورقة استخدام الاستجابة الديناميكية لعارض مستقيم لتحديد ما إذا حدث فيه ضرر أو شق. تم كجزء أول في هذا البحث، استخدام برنامج العناصر المحدودة ANSYS وتحديد الترددات الطبيعية وأشكال الاهتزازات الطبيعية للعارض في صورته السليمة بدون وجود صدع. كما تم إنشاء نموذج يحتوي على صدع وتكرار نفس التحليل. أظهرت النتائج المستخلصة حدوث تغيرات في الترددات الطبيعية مع ظهور أشكال اهتزازات جديدة للعارض. استخدام الترددات الطبيعية كتقنية لكشف حدوث ضرر أو صدع هي فعالة ولكن توجد بعض المقايضات وكان لا بد من اعتماد تقنية مختلفة. تعتمد هذه التقنية (وهي تشمل الجزء الثاني من هذا البحث) على بناء نموذج العناصر المحدودة وحساب الاستجابة الديناميكية للعوارض، ثم حساب طاقة الإجهاد لكل عنصر من العناصر على حده. تعتمد المنهجية المقدمة على جمع وتخزين البيانات الاهتزازية للعوارض في حالتها السليمة، ثم جمع نفس البيانات بعد فترة في حالة اشتباه حدوث صدوع او تشققات. يتم عندئذ استخدام مجموعتي البيانات لحساب طاقة الإجهاد في العوارض قبل حدوث الضرر، ثم حساب التغيير في طاقات الانفعال ونسبة تغيرها. تستخدم هذه النسب لتحديد العنصر/ أو العناصر التالفة في هيكل العوارض. لأجل هذا تم بناء نموذج العناصر المحدودة لعارض مستقيم، وتطبيق هذه المنهجية عليه باستخدام درجات مختلفة من سيناريوهات الضرر للتحقق من فاعلية المنهجية المتبعة ولمراقبة التغيير في السلوك الديناميكي للهياكل. أظهرت النتائج فعالية المنهجية المتبعة في هذه الورقة في الكشف عن حدوث الضرر، فضلا عن توفير معلومات عن امتداد الضرر وحجمه النسبي.

ABSTRACT

The dynamic response of a straight beam is used to identify the occurrence of damage. Apparently there is an advantage in using natural frequencies changes as an indicator for damage occurrence, but some trade-offs exist. The first part of the paper is showing these trade-offs by studying a model of an intact straight rectangular cross-section beam built in ANSYS, and identifying its modal parameters. Then a transverse crack is assumed to exist across the beam length and the modal analysis is repeated. Three scenarios of crack sizes are analyzed. These have showed a change in natural frequencies together with new mode shapes arising. Moreover, there is a relation between crack location and the dynamic modal response of the beam, mainly mode shapes. Although the analysis based only on natural frequencies is easier to conduct and the occurrence of crack is clearly observed, the trade-off is that the location of the crack is not identified. In the second part of the paper another more effective crack detection methodology is presented. It is based on calculating the strain energy of each of the finite elements used in the model. This methodology is based on collecting vibrational data of the healthy structure, i.e., before any cracks exist, then the same data are collected when cracks have supposedly occurred. The two sets of data are used to calculate ratios of modal strain energies before and after crack occurrence. These ratios are then used to locate the cracked element. The methodology is implemented in a Matlab code, where a 2-D beam finite element formulations, with two DOF per node, are used. Various degrees of damage scenarios are assumed to investigate the validity of the methodology, by observing the change in modal strain energy. The obtained results show the effectiveness of the proposed methodology in detecting crack occurrence, its location, as well as providing information on its relative severity.

KEYWORDS: Structural Health Monitoring (SHM); damage detection; Finite Element Analysis (FEA); strain energy; structural vibration.

INTRODUCTION

Beam structures are used in many structures, with varying cross-section types, and have many applications, such as in automotive industry, aerospace, and buildings, and many others. Beam structures may suffer from vibration, high stresses and deformations, and thus leading to potential failure of the structure caused by cracks. These cracks usually start small in size but then it propagates and becomes bigger, leading to catastrophic failure. Vibration monitoring has proved important to prevent structural failures, the so-called vibration-based structural health monitoring. Vibration-based damage identification methods are sometimes the alternative for the known non-destructive testing (NDT) methods. As vibrations are nonetheless a propagation of energy in the structure, thus techniques that involve energy calculation are adopted to investigate damage occurrence. Dimarogonas [1] has given a review on the vibration of cracked structures. Damage detection that is based on change of natural frequencies may be used as an indicator of damage but hardly gives information on its location [2]. Using energy techniques, namely the strain energy factor, has proved effective in localizing the damage in many cases undergoing flexural bending, as demonstrated by Stubbs et al. [3]. Nevertheless, Duffy et al. [4] has used it considering cases of torsional mode shapes, where they have proved that using torsional mode shapes could enhance the sensitivity of the strain energy method in some cases. Axial modes, when adopted, could also enhance this sensitivity, as investigated by Li et al. [5]. Kim and Stubbs [6] has investigated the use of different damage indicator based on strain energy. Method based on modal parameters have been investigated recently as in the work by Rezaei et al. [7], where the method is applied to identify damages in wind turbine blades by adopting a more complex nonlinear model. Applications in bridges are widely investigated as well, as in the work by Xu et al. [8]. Other similar technique used for damage identification is the one based on identifying the active power flowing in the structure. This technique, initially investigated by Ahmida and Arruda [9] proved to be a powerful technique and it is still under investigation. Ahmida et al. [10] have used the concept of structural intensity applied to beams. Calculating and mapping the structural intensity in a damaged structure could easily help identify the location and severity of the structural damage.

This paper covers two parts of investigations on a straight beam. In the first part, a beam is modeled via ANSYS as an intact beam. Then, various damage scenarios were investigated, each with different severity of damage, i.e., with various crack depths. The crack has a v-shaped opening, in which three different depths of crack are analyzed, demonstrating different crack severities. The relation between crack location and the dynamic modal response of the beam, mainly mode shapes, are analyzed. In the second part, a methodology for damage detection in a cantilever beam is presented. This methodology is based on the calculation of modal strain energy for the intact case of the beam, then calculate it again in case of damage. The two energies are used to estimate the location and severity of damage using a ratio of strain energy between the two cases.

PART 1: THE BEAM MODEL USING ANSYS

A steel beam, with dimensions of length=1 m, width=70 mm, thickness=10 mm, and material properties of mass density=7820 kg/m^3, and elasticity modulus=200 GPa. The beam was modeled in ANSYS-2019R2 using SOLID186, a homogenous structural solid element. This element is a higher order 3D solid element that exhibits quadratic displacement behavior. The element is defined by 20 nodes having 3 translational DOF per node, thus it is very effective to use in damage localization. Three investigations were conducted: the intact model, and two damaged scenarios of different damage locations and damage extensions.

The Intact Model

The beam was firstly modeled in its intact state, with clamped-free boundary conditions. The number of elements were increased until a converged solution was obtained. The final model consisted of a total of 5600 elements and 32517 nodes, as shown in Figure (1). In case an online crack detection system is to be employed, then this vibrational data, i.e., natural frequencies and mode shapes, have to be saved for later comparison with data of supposedly damaged structure. The algorithm to use, in this case, is the one usually called *supervised learning* algorithm, in which the data from undamaged and damaged structures are used.

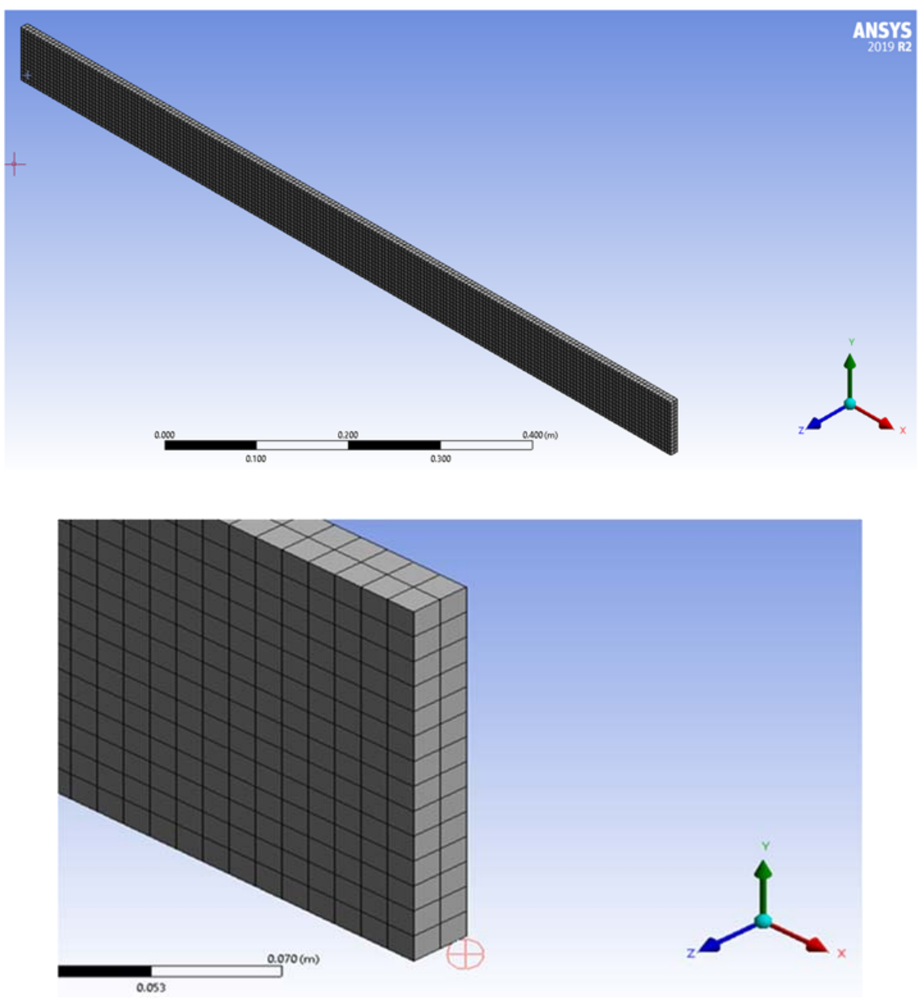


Figure 1: ANSYS model of the intact beam using SOLID186 element.

The Damaged Model

A transverse crack (v-shaped) is assumed to occur in the finite element model, located at 650mm from the clamped end of the beam. The crack has an opening width of 1mm, and three scenarios of depths, along beam width were investigated. The crack depths used were 10mm, 20mm, and 40mm, the latter being the severe case scenario. The number of elements used in the damaged beam were sufficient and compatible with the numbers used in the intact beam case. The damage of the three scenarios are shown in Figure (2).

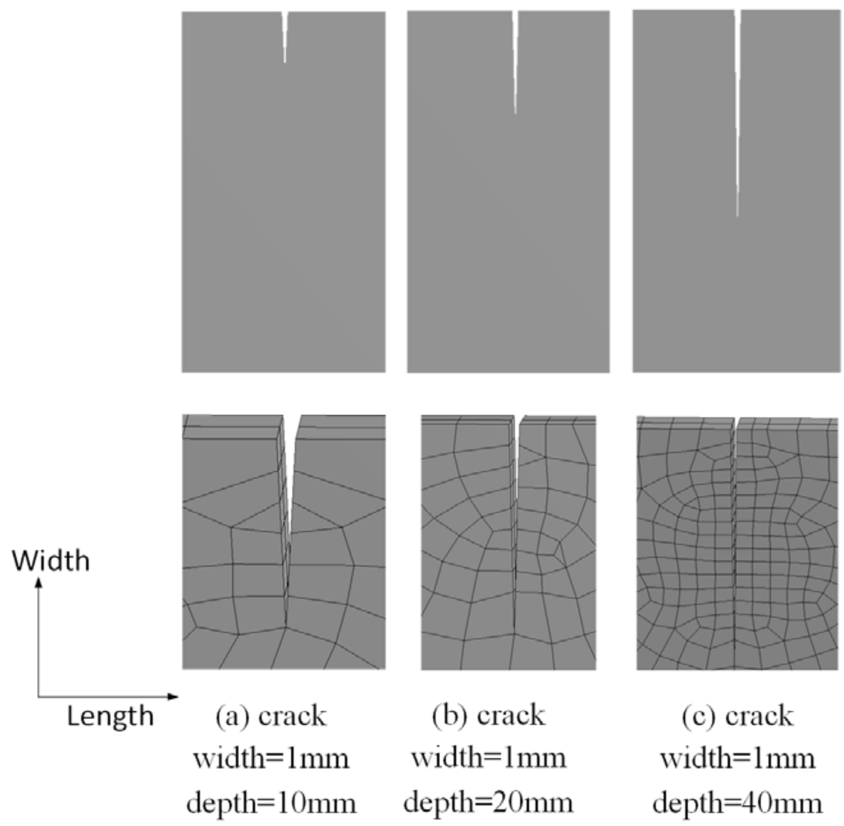


Figure 2: The three scenarios of transverse cracks across the damaged beam.

Note that the FEA mesh around the crack area has to be sufficiently refined for the numerical solution to converge. In the severe case scenario (c) of 40mm crack depth, a total of 6176 elements and 35805 nodes were used.

Results and Discussion

The first few natural frequencies and mode shapes of the intact and damaged beam are found. Because the results obtained in this part 1 are compared later on with results of part 2, the analysis in part 1 only considers flexural bending in the plane normal to beam thickness, i.e., in plane bending. All degrees of freedom in the other plane are restrained in ANSYS model. With methods based on vibrational data it may be possible to acquire damage assessment in a global manner. Experimentally, this could be accomplished using just one sensor for measurement, or if necessary just few sensors. If many sensors are used, it would be possible not just to identify the existence of damage but rather identify its location as well. In this paper we will only identify weather or not damage has occurred.

Due to the localized crack, there will be a change in natural frequency of certain mode shapes. The element with crack is expected to become more flexible, resulting from the reduction in its stiffness [11]. As a result, the values of natural frequencies will become smaller. Table (1) shows the natural frequencies of the intact beam, compared to the three damage scenarios of the beam.

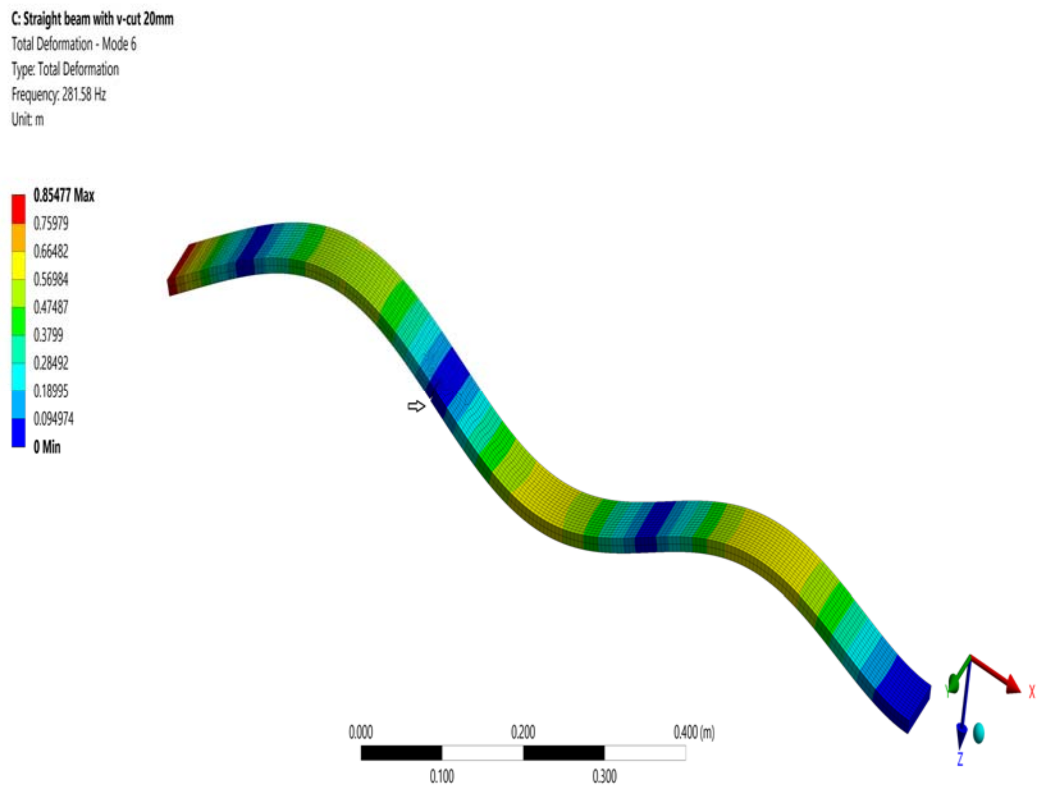
Table 1.a: Natural frequencies in Hz of the intact and damaged beam (crack depth of 10mm&20mm).

Mode No.	Intact beam	crack depth =10mm	crack depth =20mm	Remarks on mode shape
1	8.2	8.2	8.19	deflection in z-direction
2	51.38	51.27	51	deflection in z-direction
3	57.04	56.98	56.82	deflection in y-direction
4	143.82	143.45	142.51	deflection in z-direction
5	215.13	215.02	214.54	1st torsional around x-axis
6	281.72	281.69	281.58	deflection in z-direction
7	349.63	346.04	335.42	deflection in y-direction
8	465.49	464.56	462.2	deflection in z-direction
9	646.99	645.51	640.26	2nd torsional around x-axis

Table 1.b: Natural frequencies in Hz of the intact and damaged beam (crack depth of 40mm).

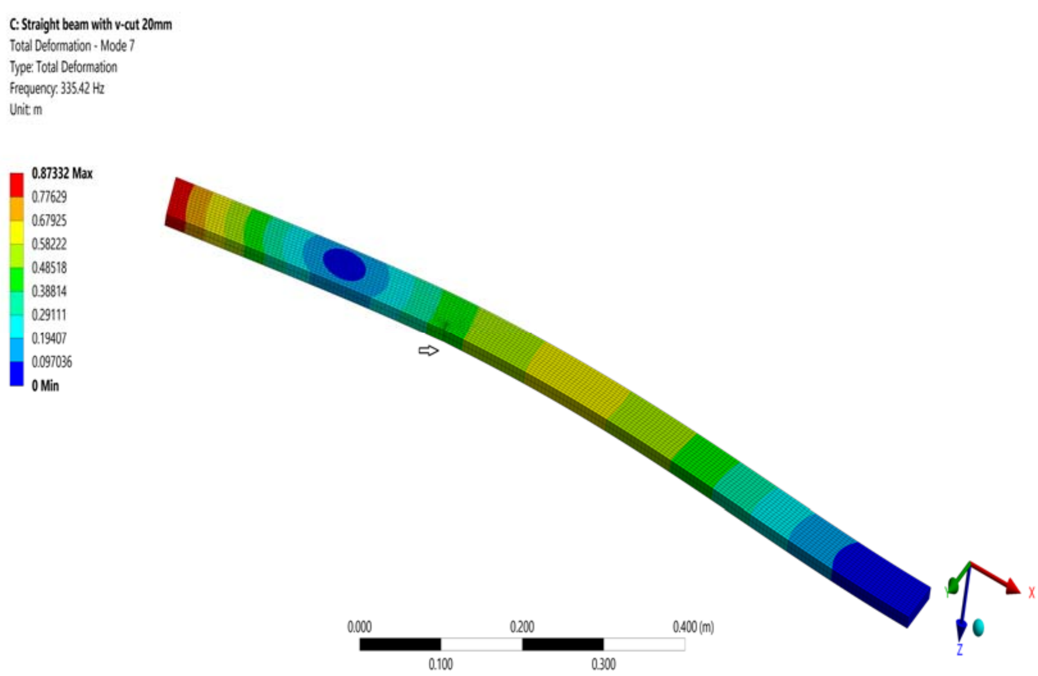
Mode No.	Intact beam	crack depth =40mm	Remarks on mode shape (only for crack depth of 40mm)
1	8.2	8.18	deflection in z-direction
2	51.38	49.81	deflection in z-direction
3	57.04	55.72	deflection in y-direction
4	143.82	138.78	deflection in z-direction
5	215.13	211.94	1st torsional around x-axis
6	281.72	280.59	deflection in y-direction
7	349.63	280.80	deflection in z-direction
8	465.49	452.72	deflection in z-direction
9	646.99	613.70	2nd torsional around x-axis

It can clearly be noticed from Table (1.a) the decrease in the natural frequencies when crack occurs, although the shape of the mode is the same, being deflection in one direction or rotation about the beam neutral axis. One interesting point to mention is the smaller change of some natural frequencies of certain mode shapes. This could be attributed to the location of the crack along the beam with respect to the mode shape. If the crack occurs in a relatively high-deformation location of a certain mode shape, it would have a higher impact on the frequency of that mode shape. For example, the case when the crack is 20mm deep (Table 1.a), the 6th mode shape is characterized by a deflection in the z-direction, as in Figure (3.a), with a frequency of 281.58 Hz. This mode shape is slightly affected by the position of the crack along the beam, where the region around the crack has lower deformations. On the other hand, the 7th and 8th mode shapes are relatively more affected by the crack position (Figures 3.b and 3.c), as the crack regions has higher deformations. Moreover, cracks that are 40mm deep has a noticeable change on the natural frequencies, where the 6 th and 7 th modes become very similar, with deflections in different directions, as demonstrated in Table (1.b).



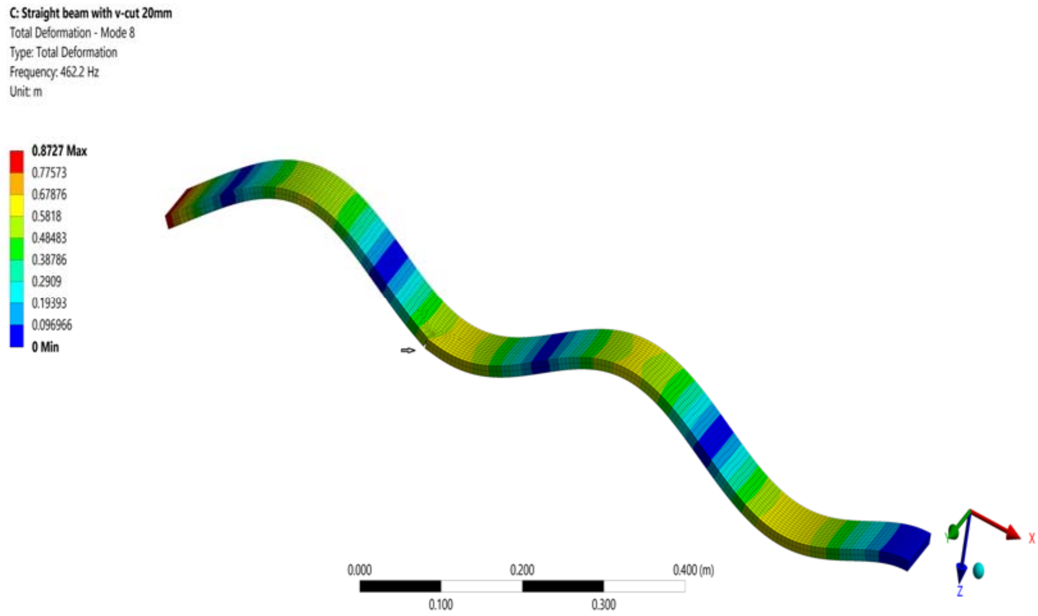
C: Straight beam with v-cut 20mm
Total Deformation - Mode 6
Type: Total Deformation
Frequency: 281.58 Hz
Unit: m
0.85477 Max
0.75979
0.66482
0.56984
0.47487
0.3799
0.28492
0.18995
0.094974
0 Min
0.000
0.200
0.400 (m)
0.100
0.300
X
Z

(a)



C: Straight beam with v-cut 20mm
Total Deformation - Mode 7
Type: Total Deformation
Frequency: 335.42 Hz
Unit: m
0.87332 Max
0.77629
0.67925
0.58222
0.48518
0.38814
0.29111
0.19407
0.097036
0 Min
0.000
0.200
0.400 (m)
0.100
0.300
X
Z

(b)



(c)

Figure 3: Some mode shapes of the beam with 20mm crack depth: (a) 6th, (b) 7th, and (c) 8th mode shapes.

It was also observed that the maximum percentage change in natural frequency is around 4% for the 20mm crack depth, see Table (2). This smaller change is due to smaller crack width used in this simulation, of width=1mm. Bigger crack widths would likely result into higher changes.

Table 2: Percentage change in natural frequencies, compared with the intact beam.

Mode No.	crack depth =10mm	crack depth =20mm	crack depth =40mm
1	0.00%	0.12%	0.24%
2	0.21%	0.74%	3.06%
3	0.11%	0.39%	2.31%
4	0.26%	0.91%	3.50%
5	0.05%	0.27%	1.48%
6	0.01%	0.05%	0.40%
7	1.03%	4.06%	19.69%
8	0.20%	0.71%	2.74%
9	0.23%	1.04%	5.15%

One other observation from these results is when the crack was bigger in depth, namely the 40mm case. In this case the 7th mode shape has a higher frequency change of about 19%. This is related to the fact that the mode shape at this frequency has changed from being a deflection in y-direction to a deflection in z-direction, which is the direction of the crack opening.

Despite the apparent advantage in using natural frequencies for damage detection, at least in early stages of diagnosis, some trade-offs do exist. Natural frequencies can be sensitive to changes in atmospheric conditions, and that could affect the diagnosis. Consequently, false alarms of structural damage or fault could occur if, for instance, an online structural health monitoring system based on natural frequency changes is adopted. A compensation algorithm or a technique would then be required to account for changes in atmospheric conditions, such as temperature and humidity for example [12].

There is a need for more precise techniques to detect damage using vibration-based methods. Therefore, the proposed methodology of using the factor of change in modal strain energy in each element or member of the beam-type structure is discussed in part 2.

PART 2: MODAL STRAIN ENERGY TECHNIQUE

A modeling framework based on modal strain energy calculations is shown. Due to damage occurrence the structural stiffness is reduced, and hence the modal strain energy of the damages member is affected. Given the case of free vibration of a dynamic system with no damping, mass matrix M and stiffness matrix K, the equation of motion in a matrix form is given by [13,14],

$$[M]\{\ddot{Q}(t)\} + [K]\{Q(t)\} = \{0\} \tag{1}$$

Thus, the eigenvalues of the dynamic system for the r-th mode shape can be given by,

$$[K]\{Q_r\} - \lambda_r[\mathrm{M}]\{Q_r\} = \{0\} \tag{2}$$

Where λ_r is the r-th eigenvalue, and Q_r is the r-th eigenvector for the structure. This equation ca be simplified to a form describing the balance of strain energy and kinetic energy,

$$\frac{1}{2}\{Q_r\}^T[K]\{Q_r\} = \frac{1}{2}\lambda_r\{Q_r\}^T[\mathrm{M}]\{Q_r\} \tag{3}$$

Thus, the modal strain energy of a structure calculated at its r-th mode shape is given by,

$$ME_{s.r} = \frac{1}{2}\{Q_r\}^T[K]\{Q_r\} \tag{4}$$

On FEA element level, the modal strain energy of the i-th element at its r-th mode shape is given by,

$$\left(me_{s,r}\right)_i = \{q_r\}_i^T[k_i]\{q_r\}_i \tag{5}$$

where $\{q_r\}_i$ - is the corresponding r-th mode shape, of the i-th element.

When damage occurs, for example in form of crack, then the modal strain energy of the damaged element can be calculated from Eq. (5) as,

$$\left(me_{s,r}^d\right)_i = \{q_r^d\}_i^T\left[k_i^d\right]\{q_r^d\}_i \tag{6}$$

As it is not possible to find the stiffness matrix $\left[k_i^d\right]$ of the damaged element, because theoretically the damaged element is not yet identified, the stiffness matrix of the healthy element $[k_i]$ is used instead. Hence, the change in modal strain energy between the intact and the damaged elements is calculated as a ratio to the modal strain energy of the intact element. This ratio is used here as an indicator of damage occurrence, and is calculated as,

$$MSECR_{ir} = \frac{(me_{s.r}{}^d)_i - (me_{s,r})_i}{(me_{s,r})_i} \tag{7}$$

where $MSECR_{ir}$ is the modal strain energy change ratio of the i-th element at the r-th mode shape, $(me_{s.r}{}^d)_i$ is the modal strain energy of the i-th damaged element on its r-th mode shape, and $\left(me_{s,r}\right)_i$ is the modal strain energy of the i-th intact element on its r-th mode shape. The quality of this indicator can be enhanced by using multiple mode shapes and calculating its average. In this case, the ratio of change in modal strain energy for the i-th element becomes,

$$MSECR_i = \frac{1}{m}\sum_{r=1}^{m} MSECR_{ir} \tag{8}$$

and the damaged element is then identified as the element with the highest value of the ratio $MSECR_i$.

Numerical Implementation

A clamped-free intact beam structure is used, with the same geometrical and material parameters. A Matlab code was written to do the simulation. The same beam data was used just for the sake of confirming the results from our Matlab model. The Bernoulli-Euler weak FEA formulations are used for the beam. The beam element stiffness and mass matrices are defined by,

$$[k_e] = \frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}$$

$$[m_e] = \frac{\rho A l}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix}$$

In this case, only simple beam deflection mode shapes in one direction (normal to beam thickness) are calculated. Only two DOFs per node are considered, translational and rotational. A total of 40 two-node elements of equal lengths were used for the solution, where it converged to the natural frequencies previously calculated in the ANSYS clamped-free example, see Table (3). Note that only deflections in z-direction now exist. Figure (4) shows the translation FRF evaluated at the free tip of the beam.

Table 3: Natural frequencies (Hz) of the intact cantilever beam: Solid elements via ANSYS vs beam element via Matlab modeling. *Only in plane deflection (in z-direction).*

Mode No.	ANSYS model	Matlab model
1	8.2	8.17
2	51.38	51.20
3	143.82	143.35
4	281.72	280.92
5	465.49	464.38
6	694.97	693.71
7	969.93	968.92

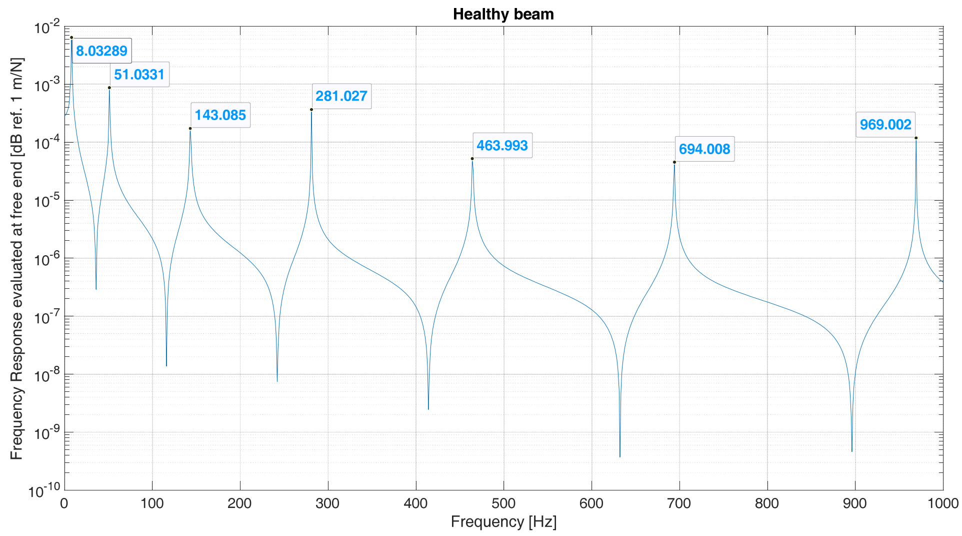
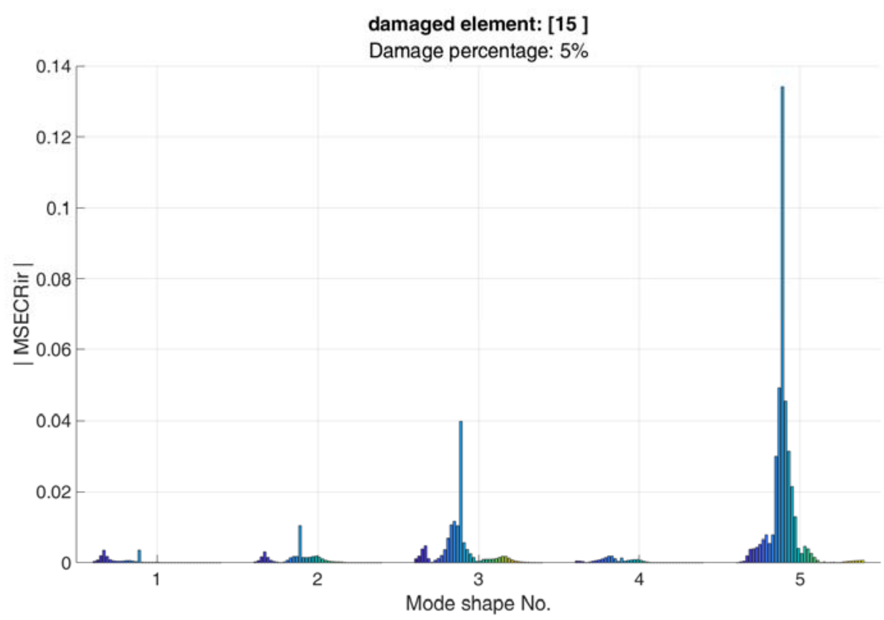


Figure 4. The frequency response of the cantilever beam evaluated at the free tip.

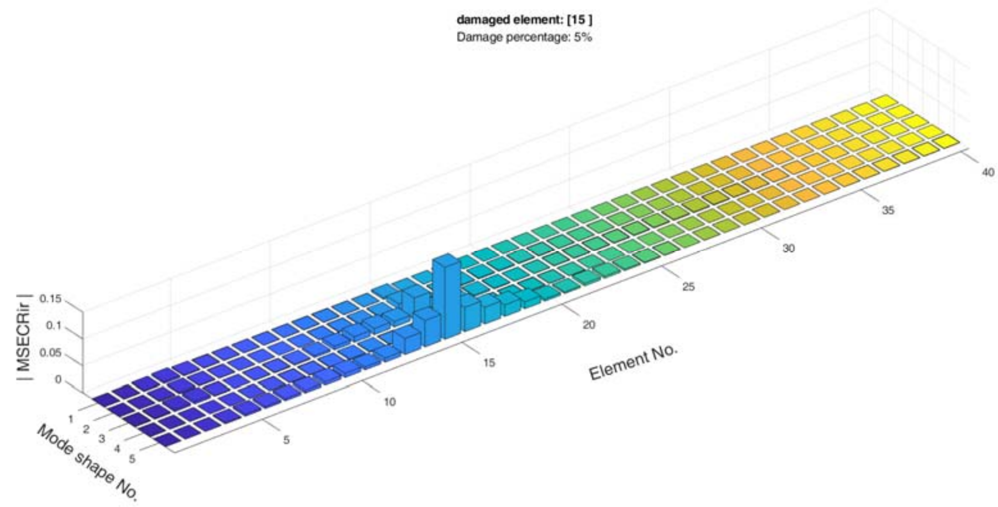
Model with Damaged Elements

A cantilevered beam model is used for the verification of MSECR methodology. A total of 40 elements are used, with 2 DOF per node. A crack is induced in the model on element number 15. This is done by reducing the element's stiffness to values varying from 5%, 10%, 20%, to 50% of the stiffness of the intact element. The changes in the stiffness are done by reducing the cross-sectional area and the moment of inertia of element 15 by these percentages.

The MSECR is calculated for each element, at each of the first five mode shapes of the structure, and for each of the cases of damage percentages. The average value of MSECR evaluated over 5 mode shapes is also calculated for every element of the mesh. The results for damage percentage of as low as 5% are shown in Figure (5a) and its 3D representation showing the element numbers in Figure (5b), and Figure (6). This is to demonstrate how sensitive is the modal strain energy method to damage occurrence.



(a)



(b)

Figure 5: The Modal Strain Energy Change Ratio, MSECRi, calculated at every mode shape. Damage is 5%. (b) the 3D representation of (a).

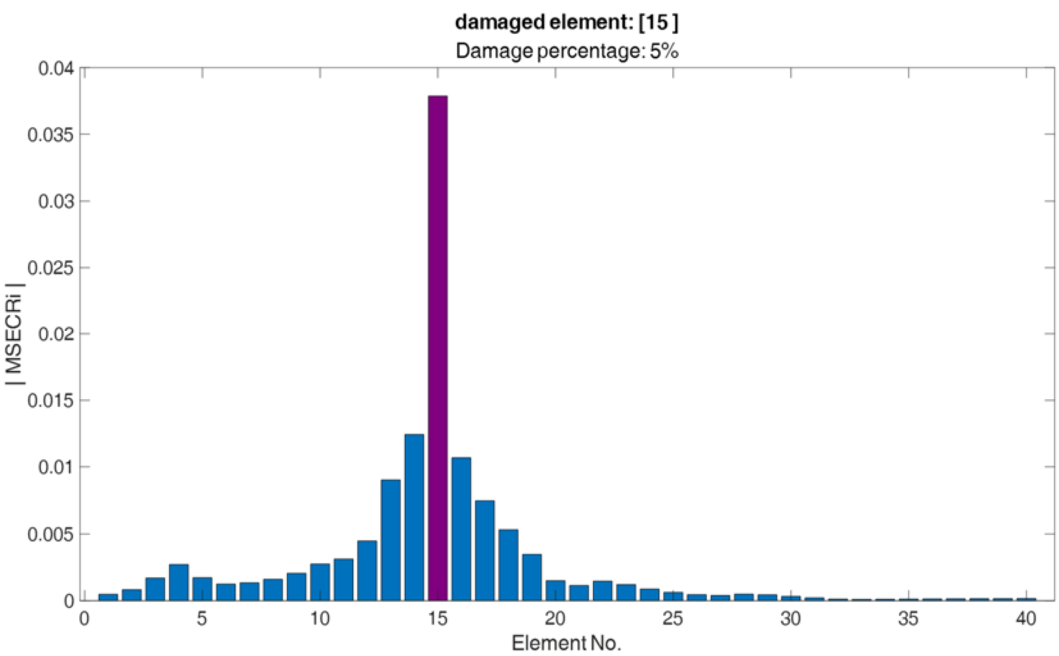


Figure 6: The Modal Strain Energy Change Ratio for every element, averaged over 5 mode shapes. Damage is 5%.

Figure (5) and Figure (6) clearly show the identification of location of the damaged element in the beam, in this case it is element 15. Note that each mode shape used in the calculation of strain energy is normalized to its maximum value. In addition, hence, the values shown are normalized to the maximum value of MSECR in each mode shape. It is noticed from Figure (5.a) that the ratio MSECR calculated at each mode always indicates the damaged element as number 15, with values becoming larger with mode shape number. Note, in Figure (6), that undamaged elements showed a slight change in MSECR, which is related to the contribution of damaged elements when averaging over 5 mode shapes. Using more mode shapes in the analysis might probably enhances the detection algorithm. Nevertheless, using higher mode shapes in the analysis requires finer finite element mesh, and an extra study should be conducted in relation to the sensitivity of using more mode shapes in damage detection. Moreover, if MSECR technique is to be used experimentally, as suggested, the measurement of higher mode shapes is not an easy task to do and becomes a key factor, and the MSECR technique becomes sensitive to noise-polluted modal data collected by measuring sensors, and might affect its robustness.

Effect on Natural Frequencies

With damage, the dynamic properties of a structure change, thus affecting its dynamic response. Because of that, the modal parameters, here regarded as the natural frequency, also change. Table (4) shows the changes for different damage scenarios in the same element number 15. It can be noticed that the change relatively increases with damage size, but not much. The change is also a function of the analyzed mode shape number and location of the damage, as previously investigated in section above.

Table 4: Natural frequencies (Hz) of the cantilever beam for different damage scenarios. (Percentage of change in parentheses).

Mode No.	intact model	damage is 5%	damage is 10%	damage is 20%	damage is 50%
1	8.17	8.16(0.1%)	8.16(0.1%)	8.15(0.2%)	8.07(1.2%)
2	51.20	51.21(0.0%)	51.23(0.2%)	51.25(0.1%)	51.12(0.2%)
3	143.35	143.38(0.0%)	143.39(0.0%)	143.37(0.0%)	142.67(0.5%)
4	280.92	280.91(0.0%)	280.91(0.0%)	280.91(0.0%)	280.87(0.0%)
5	464.38	464.35(0.0%)	464.28(0.0%)	463.99(0.1%)	460.96(0.7%)
6	693.71	693.69(0.0%)	693.64(0.0%)	693.41(0.0%)	691.00(0.4%)
7	968.92	968.92(0.0%)	968.90(0.0%)	968.83(0.0%)	968.02(0.1%)

Natural frequency change is a sensitive indicator of structural integrity, but it alone may not be sufficient to identify the location of structural damage. For example, two cracks identical sizes but at two different locations may cause the same percentage of change in natural frequency. Moreover, damage in regions of relatively higher deformations resulted in higher change in natural frequency.

CONCLUSIONS

This paper has focused on investigating crack occurrence in a straight slender beam and its effect on modal parameters, namely natural frequency and mode shape. These investigations are carried out numerically, using the commercial software ANSYS, and by coding in the Matlab package environment.

Two main conclusions are drawn. Depending on crack location and crack size, the modal parameters may change. The order of mode shapes of the intact beam may also change, and new mode shapes may arise. These mode shapes are functions of crack location and of severity of the crack itself. Because of the damage, the beam becomes more flexible (less stiffness), which results in change of natural frequencies. The other conclusion that could be drawn is that the modal strain energy shows capability to easily identify the existence, the location and the severity of structural damages. The results become sensitive if more than one damage exists in the structure. This may lead to the importance of averaging the modal strain energy over a certain number of mode shapes for better identification of damage. If MSECR technique is used with the inclusion of higher mode shapes measurements, it becomes sensitive to noise-polluted modal data collected by measuring sensors, and might affect its robustness. Further studies are recommended regarding higher mode shapes, numerically and experimentally.

NOMENCLATURE

SHM	Structural Health Monitoring	$[\boldsymbol{k_i}]$	stiffness matrix of the i^{th} intact element
FEA	Finite Element Analysis	$[\boldsymbol{k_i^d}]$	stiffness matrix of the i^{th} damaged element
DOF	Degree of Freedom	$\boldsymbol{MSECR_{ir}}$	modal strain energy change ratio of the i^{th} element at the r^{th} mode shape
FRF	Frequency Response Function	$\boldsymbol{MSECR_i}$	modal strain energy change ratio of the i^{th} element averaged over m number of modes
K	Stiffness matrix	$[\boldsymbol{k_e}]$	stiffness matrix of element *e*
M	Mass matrix	$[\boldsymbol{m_e}]$	mass matrix of element *e*
Q(t)	Generalized displacement vector	**E**	modulus of elasticity
$\boldsymbol{\ddot{Q}(t)}$	Generalized acceleration vector	**I**	moment of inertia
$\boldsymbol{Q_r}$	r^{th} eigenvector	$\boldsymbol{l}$	length of beam element
$\boldsymbol{\lambda_r}$	r^{th} eigenvalue	$\boldsymbol{\rho}$	mass density
$\boldsymbol{ME_{s.r}}$	Modal strain energy at the r^{th} mode shape	$\boldsymbol{A}$	cross-section area of beam
$\boldsymbol{(me_{s,r})_i}$	modal strain energy of the intact i^{th} element, at the r^{th} mode shape	$\boldsymbol{MSECR_{ir}}$	modal strain energy change ratio of the i^{th} element at the r^{th} mode shape
$\boldsymbol{(me_{s,r}^d)_i}$	modal strain energy of the damaged i^{th} element, at the r^{th} mode shape		
$\{\boldsymbol{q_r}\}_i$	corresponding r^{th} mode shape, of the i^{th} element		

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