NEURAL NETWORKS AUTOPILOT DESIGN FOR BALLISTIC MISSILE

Ali Mohamed Elmelhi

Electrical and Electronic Engineering Department,
Faculty of Engineering University of Tripoli, Libya
E-mail: ali_elmulhi@yahoo.com

الملخص

يتم التحكم في مسار الطيران للصواريخ البالستية آليا عن طريق نظام التوجيه والتحكم. والمسار الكلي للطيران عبارة عن مسار يجمع بين مرحلة الطيران التشغيلي ومرحلة الطيران الحر. تم في هذه الدراسة وضع النموذج الرياضى لزاوية الهجوم في مراحل مختلفة بحيث يمكن حساب متغيرات زمن توقف الاحتراق مثل سرعة الصاروخ وزاوية المسار عند الزمن النهائي لمرحلة الطيران التشغيلي. بالإضافة الى ذلك فإن العلاقة بين مدى الصاروخ البالستى ومتغيرات زمن توقف الأحتراق تم دراستها والتحقق منها. كما تم في هذه الدراسة ايضا استخدم الشبكة العصبية الأمامية لغرض دراسة استقرارية الصاروخ في الحركة الطولية، حيث تم استعمالها كطيار آلي لتوليد اشارة تحكم للتأثير على انحرفات مشغل الرفع والتي تساعد الصاروخ على المناورة ليتم تحقيق التتبع الدقيق لزاوية الهجوم المطلوبة. وللتحقق من أهداف هذه الدراسة تم عرض نتائج المحاكاة لصاروخ بالستي

ABSTRACT

Guidance and control system of the ballistic missiles normally refers to a system that automatically controls the flight trajectory. The whole flight trajectory is a combination of the powered flight stage and free flight stage. In this paper, the command angle of attack model is presented and modeled in different phases so that the burnout parameters such as path angle and missile velocity can be evaluated at the end time of the powered flight phase. In addition, the relation between the ballistic range and burnout parameters values is studied, and investigated. Furthermore, the feed forward neural network is used here to stabilize the missile in longitudinal motion. Where, it can be used as an Autopilot dynamics to generate a control signal for elevator actuator deflections which in turns cause the missile to be maneuvered, so that an accurate tracking to a command angle of attack can be satisfied. To demonstrate the objectives of this study, simulation results for a typical ballistic missile are shown at the end of this paper.

KEYWORDS: Flight Dynamics and Control; Missile Trajectory; Autopilot Design; Neural Network; Nonlinear Control Design

INTRODUCTION

A ballistic missile is a missile that has a ballistic trajectory over most of its flight path, regardless of whether or not it is a weapon-delivery vehicle. Ballistic missiles are categorized according to their range, the maximum distance measured along the surface of the earth's ellipsoid from the point of launch of a ballistic missile to the point of impact of the last element of its payload. The first issue for simulating the complete flight trajectory is to derive the six degree of freedom equations of motions [6DOf].

Some literatures give an explanation about this mathematical model derivation [1], [2], [3] and [4] which show the complete missile and aircraft nonlinear flexible and rigid dynamic models in body and wind frame axis. The burnout parameters which play an important role for satisfying the maximum range flight are also interested in [5-7]. In this paper, the mathematical model for the desired command angle of attack trajectory is presented. And based on, the values of burnout parameters such as missile velocity and flight path angle are evaluated. These parameters control the behavior and the performance of the missile flight, where they have a direct relation to the flight horizontal range and missile altitude. The evaluation of the burnout parameters for maximum flight range is investigated. Most recently, some researchers have turned to neural networks as a means of explicitly accounting for uncertain aerodynamic effects. Their on-line learning and functional approximation capabilities make neural networks an excellent candidate for this application [8-9]. As a result, the feed forward neural network has been used here for missile longitudinal Autopilot design, so that the tracking of the angle of attack nominal trajectory until the burnout time can be achieved.

In order to accomplish this study objective, this paper can be arranged as follows. First, the Quasi-3 dimensional simplified trajectory equations of motion for a ballistic missile are written; second, the angle of attack trajectory model is presented; third, the Autopilot design based feed forward neural network is discussed; finally, the computer simulation results and conclusion are included.

MISSILE MODEL

A nonlinear model is considered here in which the flight trajectory behavior motion of the missile can be described from the launching time until an impact point. And due to limited rocket structure, aerodynamic characteristic and propulsion performance data at early stage of this design. Hence it is not possible to carry out an exact trajectory of the rocket with complete equation of motion [3]. In this case, the simplified Quasi-3 dimensional trajectory model with the effect of earth rotation is applied here [10], where the following simplifications are taken into account:

- Lateral motion is weak [along z direction].
- Pitching motion is in equilibrium.
- The earth is spherical.

The geometrical diagram which shows the configuration of the missile in the launch frame is shown in Figure (1). The main challenging associated with powered flight phase intercept is the short time associated, typically between 50 and 300 seconds depending on the missile's range and propellant type. This flight phase is starting from the launching until burnout time [shut-off engine time]. In this phase, the missile is under the influence of the guidance and control which are designed and located respectively in the outer and inner loops. Where, in this study, the guidance is represented by the external command angle of attack $\alpha_c\ (t)$ and control law is replaced by an Autopilot designed based on neural network. To see the trajectory behavior of the missile during whole time of flight, the following nonlinear model is simulated:

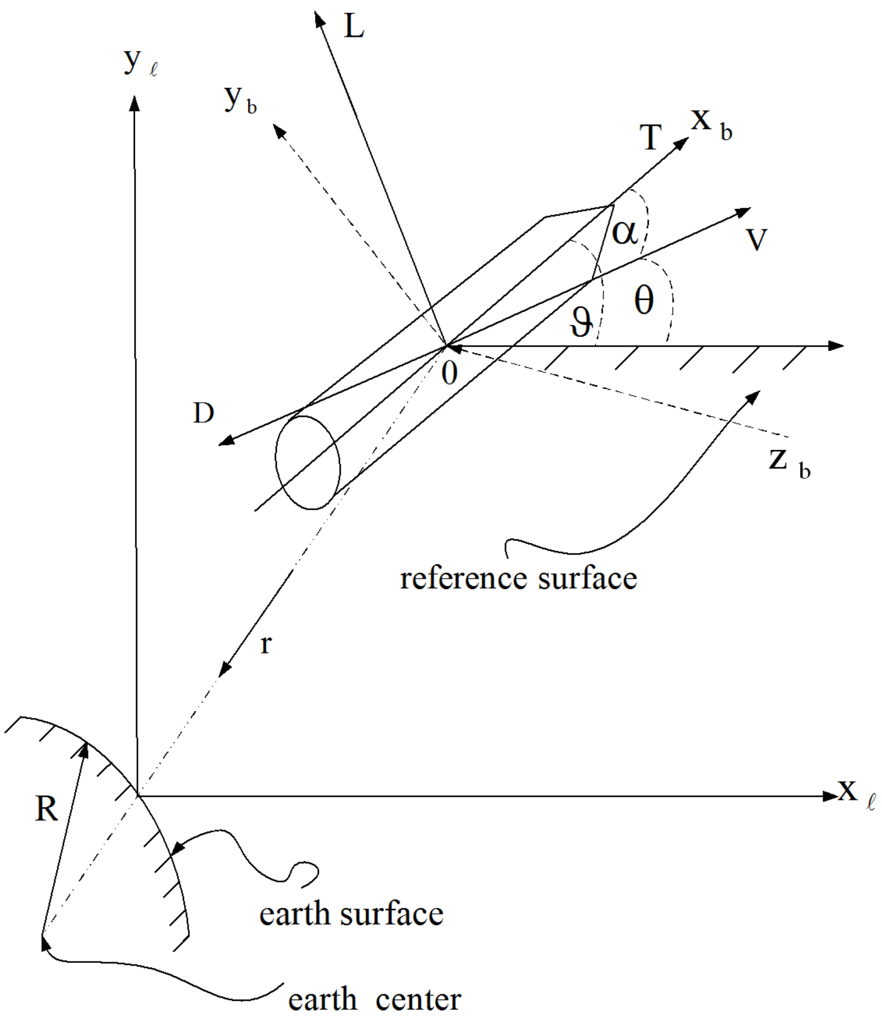


Figure 1: Missile configuration in pitch plane

$$\frac{dV_{x_l}}{dt} = \frac{1}{m_a}\left(F_{xb}cos\theta - F_{yb}sin\theta\right) + g_{x_l} + f_{cor x_l} \quad (1)$$

$$\frac{dV_{y_l}}{dt} = \frac{1}{m_a}\left(F_{xb}sin\theta + F_{yb}cos\theta\right) + g_{y_l} + f_{cor y_l} \quad (2)$$

$$\dot{w}_z = M_z^{w_z} w_z + M_z^{\alpha} \propto + M_z^{\delta_\vartheta}\delta_\vartheta \quad (3)$$

$$w_z = \vartheta \quad (4)$$

$$\frac{dx_l}{dt} = V_{x_l} \quad (5)$$

$$\frac{dy_l}{dt} = V_{y_l} \quad (6)$$

$$\frac{dm_a}{dt} = -\mu_p \quad (7)$$

Where the total forces in x_b and y_b axes are given respectively as follows.

$$F_{xb} = T - Dcos(\alpha) + L\, sin(\alpha) \quad (8)$$

$$F_{yb} = D\sin(\alpha) + L\cos(\alpha) \tag{9}$$

And

$$g_{x_l} = -\left(\frac{\mu}{r^3}\right)x_l \tag{10}$$

$$g_{y_l} = -\left(\frac{\mu}{r^3}\right)(y_l + R) \tag{11}$$

$$D = 0.5C_D\rho V^2 S_a \tag{12}$$

$$L = 0.5C_L\rho V^2 S_a \tag{13}$$

Where μ is the product of the earth mass and earth gravitational constant $(3.986005 \times 10^{14} \ \ \mathrm{m}^3/sec^2)$.

$$r^2 = x_l^2 + (y_l + R)^2 \tag{14}$$

$$V = \sqrt{V_{x_l}^2 + V_{y_l}^2} \tag{15}$$

$$M_z^{w_z} = m_z^{w_z} QS_a l^2/V \tag{16}$$

$$M_z^{\alpha} = m_z^{\alpha} QS_a l \tag{17}$$

$$M_z^{\delta_\vartheta} = \mathrm{T}(\mathrm{l_T} - \mathrm{x_z})/\sqrt{2} \tag{18}$$

$$f_{cor.x_l} = 2V_{y_l}\omega_{E.z_l} \tag{19}$$

$$f_{cor.y_l} = -2V_{x_l}\omega_{E.z_l} \tag{20}$$

$$tan(\theta) = \frac{V_{y_l}}{V_{x_l}} \tag{21}$$

And due to assumption of a weak motion in z direction, the equivalent force equation is only evaluated from the corrillos effect as written below.

$$\frac{dV_{z_l}}{dt} = f_{cor.z_l} \tag{22}$$

$$\frac{dz_l}{dt} = V_{z_l} \tag{23}$$

Where

$$f_{cor.z_l} = 2\left(-V_{y_l}\omega_{E.x_l} + V_{x_l}\omega_{E.y_l}\right) \tag{24}$$

$$\omega_{E.xl} = \omega_e cos(B_l)cos(A_l) \tag{25}$$

$$\omega_{E.yl} = \omega_e sin(B_l) \tag{26}$$

$$\omega_{E.zl} = \omega_e sin(A_l)cos(B_l) \tag{27}$$

Geometrical relation

$$\vartheta = \theta + \alpha \tag{28}$$

The above model is carried out in the powered and free flight phases, where in the second phase, the missile continue to fly as projectile with initial flight parameters given by burnout flight path angle and velocity as will be mentioned later in the simulation results.

ANGLE OF ATTACK TRAJECTORY MODEL

The design of this trajectory model is an important issue for the powered flight performance. This is because the command angle of attack which can achieve the burnout parameters for maximum flight range can be evaluated. And this model is defined independently according to the following distinct time periods:

- ***Vertical ascend*** *[* $0\ to\ t_1$ *]*

In this time duration of flight, the following parameters are selected:

$\vartheta = 90\ ,\ \ \alpha_c(t) = 0\ , \left({d\theta}/_{dt}\right) = 0\ , \theta = 90$

The minimum permissible duration of this flight time is determined by the conditions of launch safety. The duration of this phase is selected as short as possible because the greater it is, the steeper is the trajectory (the velocity losses in overcoming gravity are increased) and it is more difficult to accomplish turning of the missile in the subsequent phase period. Normally the velocity of the missile corresponding to this phase is approximately about$30\ to\ 40\ m/sec$.

- ***Turning by air force and gravity*** *[* $t_1\ to\ t_2$ *]*

After the previous initial corrective phase, the missile enters into the turning phase. Where, the missile has to accomplish a comparatively large turning in small duration of time. Therefore the effect of this region on flight performance and shell body of missile is large. And the angle of attack model in this phase has to be adjusted in order to achieve suitable burnout parameters in which the maximum range distance can be satisfied. In this case, the angle of attack parameters can be chosen as follows.
$(d\vartheta/dt) < 0\ , \alpha_c(t) < 0\ , (d\theta/dt) < 0$

Where the variation of the angle of attack $\propto_c(t)$ can be evaluated based on the following introduced exponential formula:

$$\alpha_c(t) = -4a_m\,\Gamma(t)(1-\Gamma(t)) \qquad (29)$$

Where

$$\Gamma(t) = e^{(-\xi(t-t_1))} \qquad (30)$$

With $a_m = |\alpha_c(t)|_{max}$ and ξ is a constant.

And the following boundary condition has to be satisfied

$\propto_c(t_1) = 0\ , \propto_c(t_2)$ must be very small

- ***Turning by gravity*** *[* $t \geq t_2$ *]*

In this case, the following parameters are selected where the angle of attack is zero and the missile will turn only due to effect of gravity force:

$(d\vartheta/dt) < 0\ , \propto_c(t) = 0\ , (d\theta/dt) < 0$

AUTOPILOT DESIGN

Traditional Autopilot design requires an accurate aerodynamic model and relies on a gain schedule to account for system nonlinearities [11, 12]. In this section, the

Autopilot model for stabilizing the considered missile in the powered phase is illustrated. It is formulated based on the feed forward neural network control structure. This design methodology represents a nonlinear controller which is trained online to generate a control signal $u_{nn}(t)$ instantly for actuator deflections. Figure (2-a) shows the schematic diagram for pitch Autopilot design. It is seen that, the error signal $\boldsymbol{e}(t)$ will adapt the neural network structure by tuning the weighting parameters until the input objective angle of attack command $\propto_c (t)$ is satisfied. And it should be noted that, in the following discussions and descriptions, all the vectors are denoted by bold letters.

The general structure of the multilayer feed forward neural network is shown in Figure (2-b), where the input layer is related to the external inputs represented by vector $\boldsymbol{P}$. Each layer is related to the next layer massively by the weighting matrix $\boldsymbol{W}^m$ with m is the layer number, and the output of each layer is the input of the next one. This general structure is assumed to have the same number of neurons denoted by h, and each neuron have bias input given by $\boldsymbol{b}_h^m$ where h is the number of neurons in each layer. The activation functions $\boldsymbol{f}^m$ are assumed to be linear or nonlinear functions.

In this design, the learning process of the considered neural network is carried out by the Backpropagation algorithm which can be described as follows [13, 14].

The first step is to propagate the input with vector $\boldsymbol{P}$ forward through the network.

$$\boldsymbol{a}^0 = \boldsymbol{P} \tag{31}$$

The output in each hidden layer is given by

$$\boldsymbol{a}^{m+1} = \boldsymbol{f}^{m+1}(\boldsymbol{W}^{m+1}\boldsymbol{a}^m + \boldsymbol{b}^{m+1}) \qquad m = 1,2 \quad ,M-1 \tag{32}$$

Where, M represent the number of layers in network, $\boldsymbol{f}$ is a vector of the activation function selected by the designer and $\boldsymbol{b}$ is the bias vector.

And the output of the neurons in the last layer (output layer) is obtained by

$$\boldsymbol{a} = \boldsymbol{a}^M \tag{33}$$

In order to learn the weights and biases for the considered neural network, the following two equations are given respectively.

$$\boldsymbol{W}^m(k+1) = \boldsymbol{W}^m(k) - \eta \boldsymbol{S}^m(\boldsymbol{a}^{m-1})^T \tag{34}$$

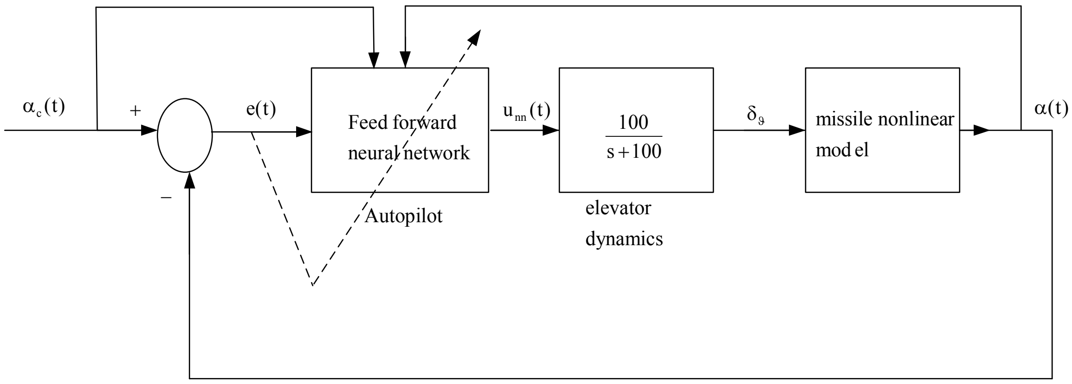
$$\boldsymbol{b}^m(k+1) = \boldsymbol{b}^m(k) - \eta \boldsymbol{S}^m \tag{35}$$

Where η is the learning rate and the sensitivities $\boldsymbol{S}^M$ and $\boldsymbol{S}^m$ can be propagated backward through the network by the following two relations:

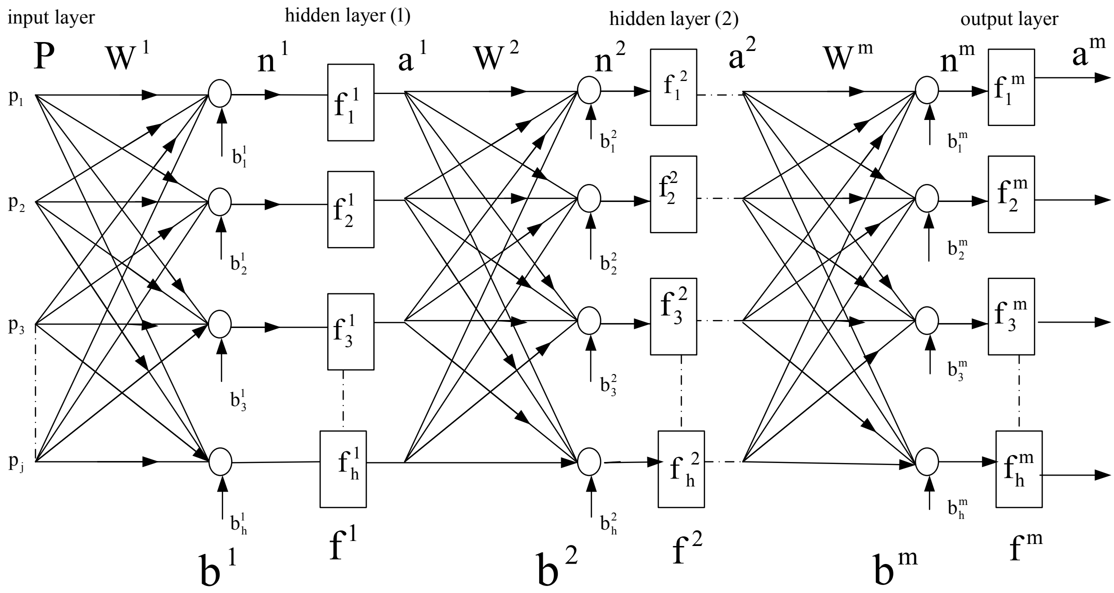
$$\boldsymbol{S}^M = -2\dot{\boldsymbol{f}}^M(\boldsymbol{n}^M)(\boldsymbol{t}_g - \boldsymbol{a}) \tag{36}$$

$$\boldsymbol{S}^m = \dot{\boldsymbol{f}}^m(\boldsymbol{n}^m)(\boldsymbol{W}^{m+1})^T\boldsymbol{S}^{m+1} \ , \quad \text{for } m = 1,2,\ldots..M-1 \tag{37}$$

With vector $\boldsymbol{n}$ denotes the output of the neurons and inputs to the equivalent activation functions in each network layer, and $\boldsymbol{t}_g$ is the target vector.



(a)



(b)

Figure 2: (a) Pitch autopilot control structure (b) Multi layer feed forward neural network structure

SIMULATION OF RESULTS AND DISCUSSION

The computer simulation is carried out by solving the nonlinear differential equations based on some typical geometrical, propulsive and structure data for missile V2 [6]. To illustrate the useful of this study, the following simulation results are performed using MATLAB/Simulink**.** In this simulation, the following initial conditions at the starting time of the powered flight phase are considered:

$T = 280000\,N$, $\mu_p = 12980\,kg$, $V_{xl} = 0$, $V_{yl} = 0$ $V_{zl} = 0$, $w_z = 0$, $\theta = 90\,deg$,

$\vartheta = 90\,deg$, $\alpha = 0$, $\delta_\vartheta = 0$, $x_l = 0$, $y_l = 0$, $\omega_e = 7.29 \times 10^{-5}\,deg/sec$.

Initially, the missile is launching vertically, and after few seconds, it turns aerodynamically by an elevator actuator deflections which are caused due to the influence of the control signal $u_{nn}(t)$. This is only carried out until the time of burnout is reached. After that, the missile enters into the free flight phase in which the following flight conditions are substituted:

$T = 0\ N$, μ_p =0 kg , $u_{nn} = 0$.

And in order to accomplish the useful of this study analysis, the following three subsections are discussed:

Neural Network for Autopilot Design

As mentioned before, the nonlinear Autopilot for stabilizing a longitudinal motion of the considered missile is designed based on the multi layered feed forward neural network shown in Figure (2). In this design work, the network is composed of three layers, the input layer is directly related to the external input vector . The inputs in this vector are connected massively to the hidden layer via the input weighting functions described by matrix $\boldsymbol{W}^1$. The connection between the hidden layer and output layer can be achieved by the weighting matrix $\boldsymbol{W}^2$. There are three neurons in this layer $[h = 3]$with tan-sigmoid functions denoted respectively by f_1^1 , f_2^1 and f_3^1, and a single neuron in output layer with the same tan-sigmoid function f^2. Where the activation nonlinear sigmoid functions is expressed by the following exponential formula:

$$f(n^m) = \left(e^{(n^m)} - e^{-(n^m)}\right)/\left(e^{(n^m)} + e^{-(n^m)}\right) \quad (38)$$

In the current scenario, the objective is to track an external desired input command angle of attack $\propto_c (t)$ while holding lateral displacement near zero. To test the effectiveness of this design, the block diagram shown in Figure (2-a) is simulated. Where the elevator command deflection δ_ϑ is related to the control signal $u_{nn}(t)$ by the following first order differential equation:

$$\dot{\delta}_\vartheta(t) + 100\delta_\vartheta(t) = 100u_{nn}(t) \quad (39)$$

The backpropagation algorithm mentioned previously is formulated according to this designed structure of multi layer feed forward neural network. And it can be summarized as following:

The external inputs are described by

$\boldsymbol{a}^0 = \boldsymbol{P}$

Where

$\boldsymbol{P} = [\alpha_c(t), e(t), \alpha(t)]$

With error $e(t)$ given by

$e(t) = \alpha_c(t) - \alpha(t)$

The sensitivities for both layer $\boldsymbol{S}^1$ and $\boldsymbol{S}^2$ can be evaluated as follows.

$M = 2\,,\; m = 1{,}2$

$S^2 = -2\dot{f}^2(n^2)e(t)$
$\boldsymbol{S}^1 = \dot{\boldsymbol{f}}^1(\boldsymbol{n}^1)(\boldsymbol{W}^2)^T\boldsymbol{S}^2$
Where both $\boldsymbol{n}^1$ and $\boldsymbol{n}^2$ are the net inputs to the first and second layer respectively which can be described by

$\boldsymbol{n}^1 = \boldsymbol{W}^1\boldsymbol{P}$
$n^2 = \boldsymbol{W}^2\boldsymbol{a}^1$
$\boldsymbol{a}^1 = \boldsymbol{f}(\boldsymbol{n}^1)$
And the output of the network is obtained from the output layer as

$a^2 = f(n^2)$
This output represents the control signal $u_{nn}(t)$.
Accordingly, the biases and weighting matrices can be updated as following:
For output layer

$\boldsymbol{W}^2(k+1) = \boldsymbol{W}^2(k) - \eta S^2(\boldsymbol{a}^1)^T$

$b^2(k+1) = b^2(k) - \eta S^2$

For hidden layer

$\boldsymbol{W}^1(k+1) = \boldsymbol{W}^1(k) - \eta\boldsymbol{S}^1(\boldsymbol{a}^O)^T$

$\boldsymbol{b}^1(k+1) = \boldsymbol{b}^1(k) - \eta\boldsymbol{S}^1$

To carry out the simulation of this learning algorithm, the following random initial conditions are selected:

$$\boldsymbol{W}^1(0) = \begin{bmatrix} 0.4501 & 0.2621 & 0.1154 \\ -0.2689 & -0.0435 & 0.2919 \\ 0.1068 & -0.4815 & 0.4218 \end{bmatrix} ,$$

$$\boldsymbol{W}^2(0) = [\; -0.4421 \quad 0.3132 \quad -0.3611]$$

$$b^1(0) = 0\; ; \boldsymbol{b}^2(0) = [0; 0; 0]$$

With Backpropagation learning rate $(\eta = 0.1)$.

From the solution of (39), the elevator deflection response during nonlinear simulation is obtained as shown in Figure (3). This deflection trajectory behavior is obtained due to effect of the Autopilot control signal $u_{nn}(t)$ shown in Figure (4). As a result, the missile rigid body is maneuvered so that the nominal command angle of attack trajectory $\alpha_c(t)$ can be tracked accurately as observed in Figure.5, in which the solid line related to $\alpha_c(t)$ and dotted line related to the actual output angle of attack $\alpha(t)$.

It should be noted that, in comparing to the other simulation results, the time scale for Figure (3), Figure (4) and Figure (5) is only limited to 15 sec. This is because the variation of their related physical parameters are occur in short period of time and also to make the behavior trajectories of these parameters to be more obvious.

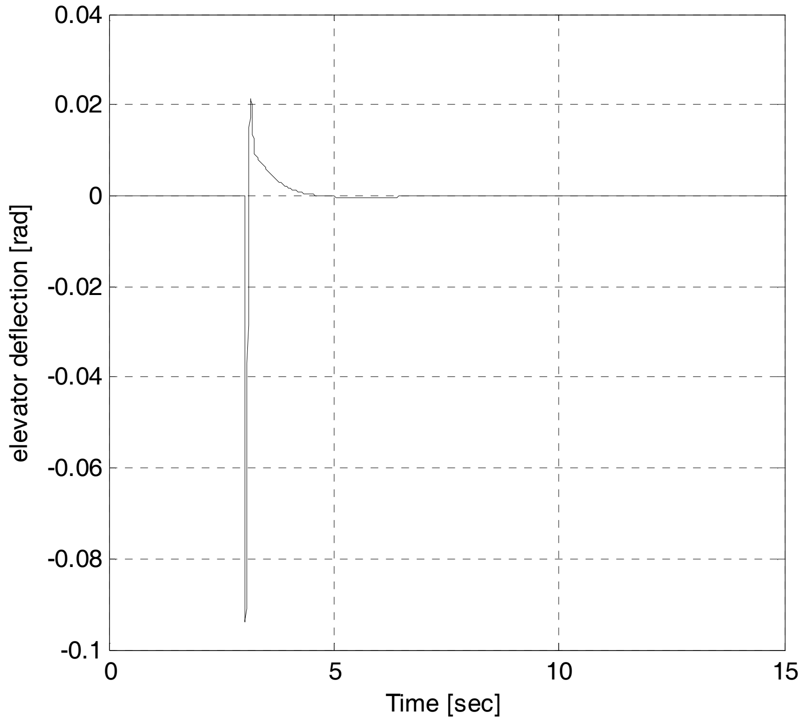


Figure 3: Elevator angle deflection $\delta_{\vartheta}(t)$

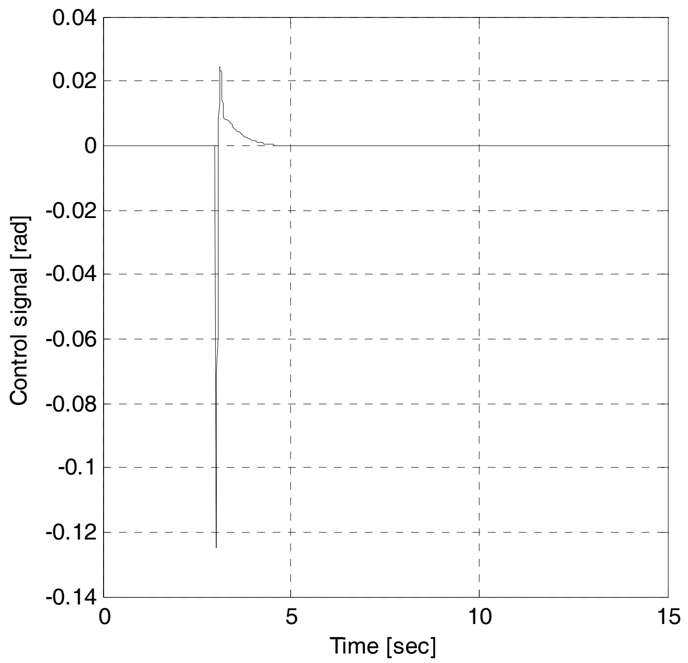


Figure 4: Control signal $u_{nn}(t)$

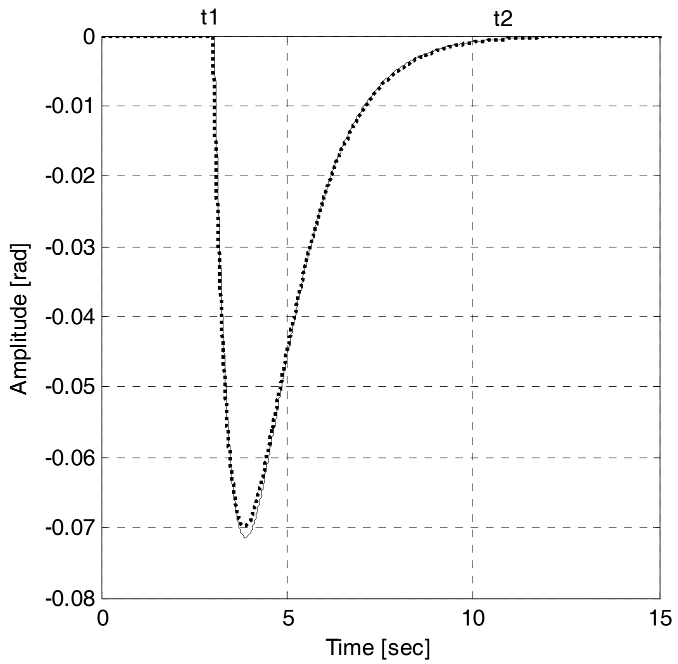


Figure 5: $\alpha(t)$ and $\alpha_c(t)$ versus time

Burnout Parameters Simulation

In this simulation analysis, the flight path and missile velocity burnout parameters are evaluated and drawn. These results are an essential ingredient of the analysis of range trajectories on a rotating earth frame.

Figure (6) and Figure (7) show respectively the trajectories behavior for both the flight path angle and missile velocity. Which are indicated by different trajectory shapes in order to make an equivalent relation and analysis with respect to an angle of attack trajectories behavior shown in Figure (8). In addition, Table (1) shows this tradeoff relation numerically at burnout time $t = 64\ sec$. In which the angle of attack model factors such as t_1 , ξ and a_m are selected randomly and then change their values independently to seek the burnout parameters which may achieve the maximum flight range.

Table 1: Burnout and Angle of Attack Parameters

Angle of attack parameters			burnout time t_b	Burnout parameters		Line shape
t_1	ξ	a_m		V	θ_p	
3 sec	0.3	4 deg	64 sec	1579.904 m/sec	55.02 deg	Dashed line
6 sec	0.3	4 deg	64 sec	1559.46 m/sec	73.08 deg	Dotted line
3 sec	0.3	6 deg	64 sec	1597.09 m/sec	39.59 deg	Solid line
3 sec	0.8	4 deg	64 sec	1567.28 m/sec	65.45 deg	Dashed dotted line

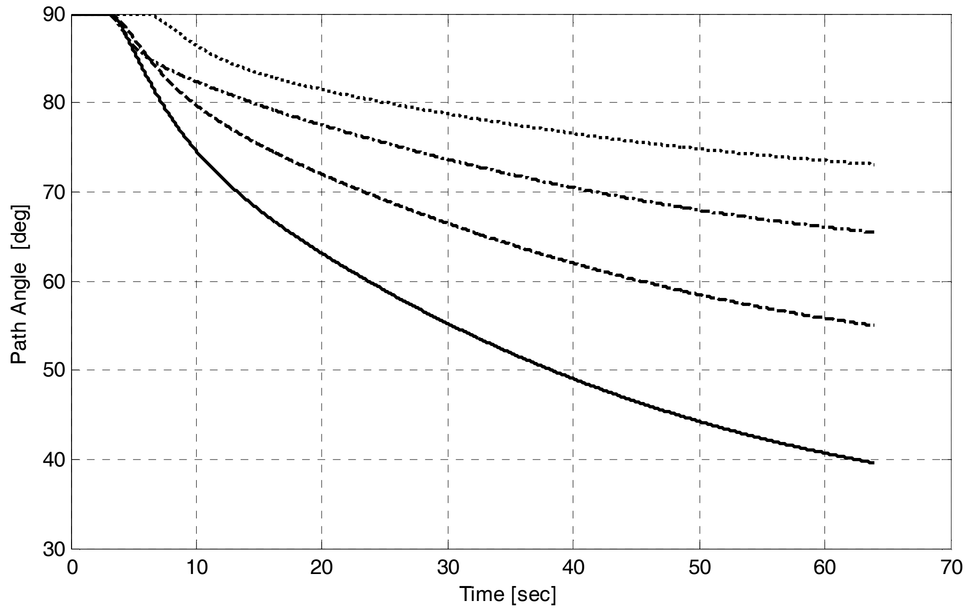


Figure 6: Flight path angle trajectories

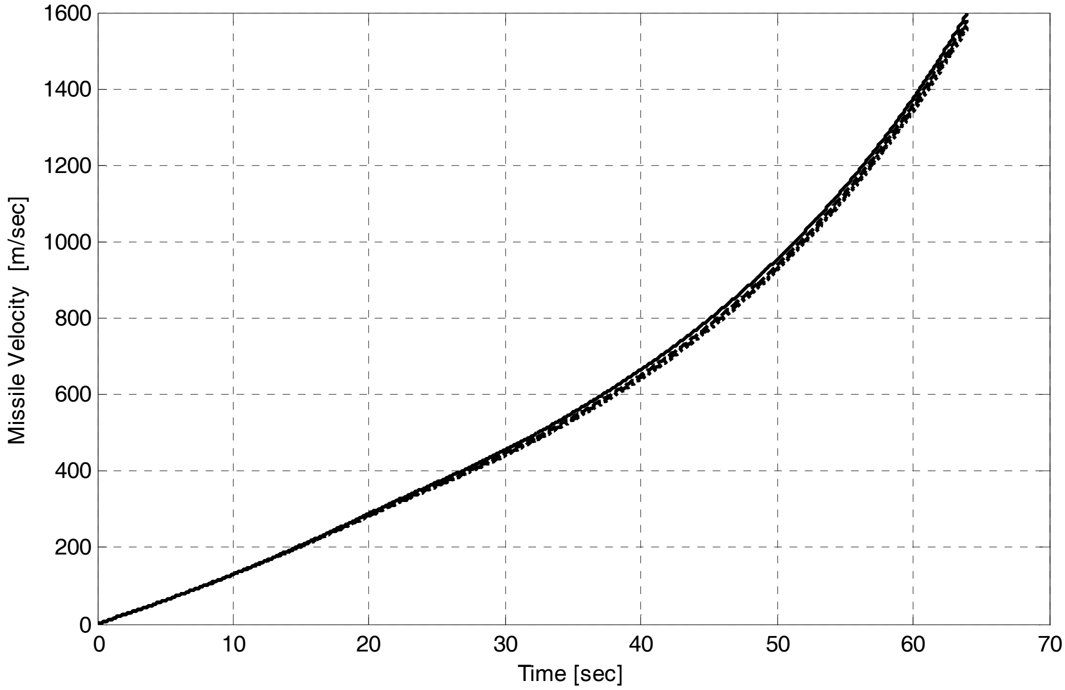


Figure 7: Velocity trajectories

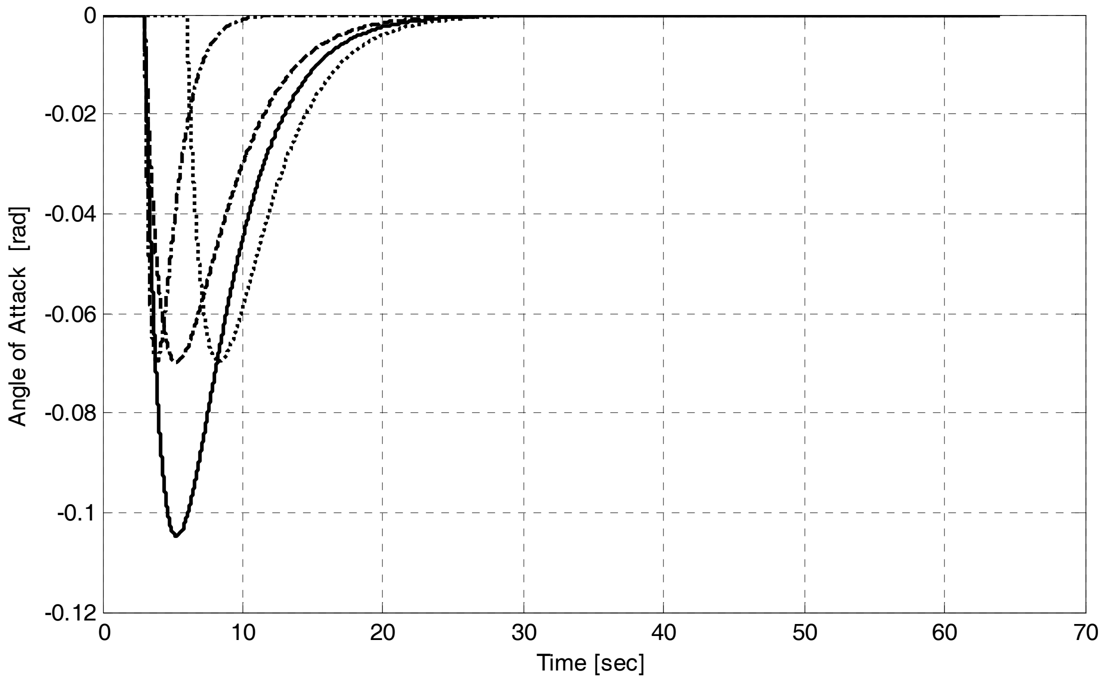


Figure 8: Command angle of attack trajectories

Trajectory Simulation

The burnout parameters which have been obtained previously are the initial conditions for the missile in free flight phase. Consequently, the range is evaluated after this phase in which all objects follow ballistic trajectories under the sole influence of Earth's gravitational field. And noteworthy because it is the longest phase of a missile's flight thereby providing more time for observing and reacting to the threat. Table (2) shows the relation between the flight total range and the burnout parameters. It is seen that, the maximum range can be achieved in case of the highest velocity and lowest path angle. Similarly, this can be demonstrated from the solid line trajectories shown in Figure (6), Figure (7) and Figure (9). Where, in Figure (9) the relation between the missile altitude and horizontal ballistic range is shown at distinct values of designed burnout parameters. It is seen that, the trajectory with solid line represents the maximum missile flight range [313.25 km]. Which corresponding to the maximum velocity and minimum path angle given respectively as $1597.09\ m/sec$ and $39.59\ deg$.

Table 2: Trajectory Range Versus Burnout Parameters

Burnout parameters		Ballistic Flight Range	Line shape
V	θ_p		
1579.904 m/sec	55.02 deg	277.89 km	Dashed line
1559.46 m/sec	73.08 deg	153.51 km	Dotted line
1597.09 m/sec	39.59 deg	313.25 km	Solid line
1567.28 m/sec	65.45 deg	215.43 km	Dashed dotted line

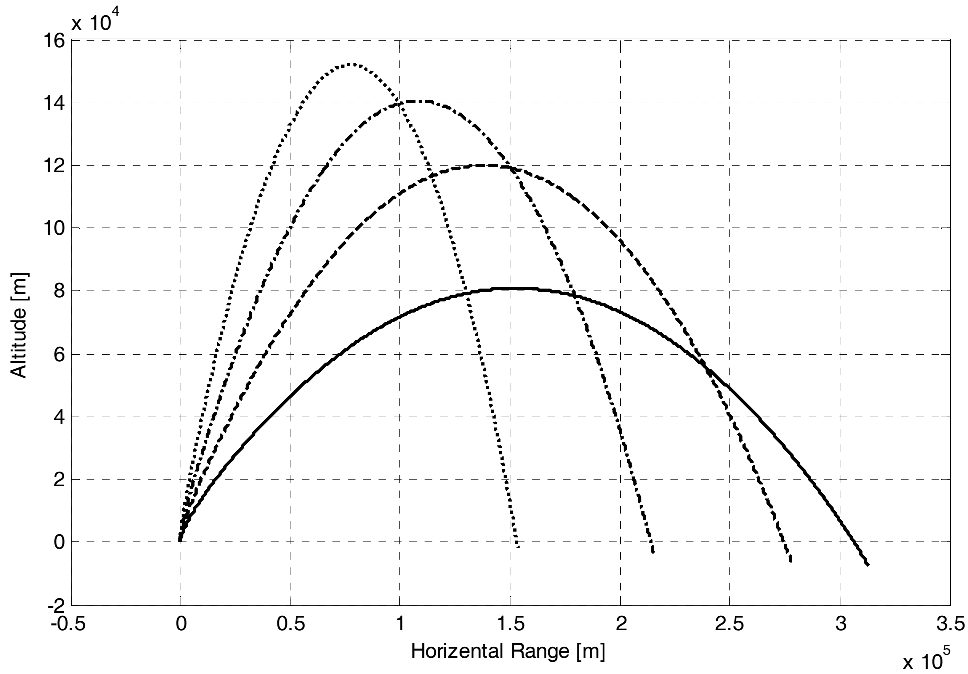


Figure 6: Missile flight trajectory

CONCLUSION

The simplified missile Quasi-3 dimensional equations are considered and the angle of attack model for a desired input command trajectory is presented. In addition, the feed forward neural network based on Backpropagation algorithm has been used as a nonlinear controller. One advantage of the neural-network-based approach is that it eliminates the time-consuming process of model linearization and designing a different Autopilot at each of numerous flight conditions called gain scheduling. Additionally, the use of neural networks enables the nonlinear controller to effectively adapt on-line so that the missile uncertain aerodynamic data might be addressed. These results encourage further investigation of neural networks for missile Autopilot design.

The designed Autopilot via neural network was successful in a achieving a desired command angle of attack and in consequently different burnout parameters have been obtained. Throughout the simulation studies, it was concluded that, after investigation of the evaluated burnout parameters, the missile maximum velocity and minimum path angle are the required conditions for maximum flight range.

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NOMENCLATURES

m_a	Vehicle mass [*kg*].
g	Earth gravity [m/sec^2].
$x_l\ y_l\ z_l$	Missile launch frame axes.
$x_b\ y_b\ z_b$	Missile body frame axes.
$V_{x_l}, V_{y_l}, V_{z_l}$	Missile velocity in x_l, y_l and z_l axes [m/*sec*].
V	Vehicle velocity [m/*sec*].
θ	Flight path angle [*deg*].
α	Angle of attack [*deg*].
ϑ	Pitch angle [*deg*].
δ_ϑ	Elevator deflection [*deg*].
w_z	Pitch angular velocity [*deg/sec*].
μ_p;	Mass propellant flow rate $(\boldsymbol{kg/sec})$
T	Effective thrust in Newton [*N*].
R	Earth radius [m].

C_D , C_L	Aerodynamic drag and lift coefficient.
S_a	Surface area [m^2].
ρ;	**Air density** $(kg - m^{-3})$.
r	**Radial distance [m].**
$m_z^{w_z}$	Moment coefficient due to pitch rate.
m_z^{α}	Moment coefficient due to angle of attack.
Q	Dynamic pressure [$\mathbf{kg/m^3}$].
l	Reference length [m].
L_T;	**Total missile length** (m).
x_z;	**D**istance from theoretical tip of missile center of Mass(m).
ω_e	Earth angular velocity rotation $(deg/sec]$.
A_l , B_l	Geographical longitude and Azimuth angles [*deg*].