EFFECT OF THE WAKE BEHIND WIND ROTOR ON OPTIMUM ENERGY OUTPUT OF WIND FARMS

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الملخص

هذه الدراسة تتعامل مع نمذجة تأثير المخر (wake effect) على الطاقة المستخلصة من مزارع الرياح، وتغطي تأثير المخر الناتج من تداخل العضو الدّوار الأمامي مع العضوين يمينهٍ ويسارهٍ في حال توربينات الوسط، ومع أحدهما في حال التوربينات الحدية. ولقد تَم تطوير نموذجا رياضيا أحادي المخر بالاعتماد على الوصف الخطي، وذلك لحساب سرعة الرياح داخل منطقة المخر عند أي مسافة خلف العضو الدّوار في مزرعة الرياح. وتَم استخدام نوعين من التوربينات بقطرين مختلفين (62 م و 100 م) لتقدير تأثير المخر على الطاقة المنتجة من مزارع الرياح. تَم إعداد توزيعات مختلفة لمزرعة الرياح، منها: 3×3 (صفوف×أعمدة)، 4×4، 6×6، 1×16، 16×1، 2×8 و 8×2، وحساب الطاقة المستخلصة لكل حالة لاختيار التوزيع الأمثل من بينها، وبالتالي مقارنة هذه التوزيعات بالتوزيع الأمثل. يوجد انخفاض في الطاقة السنوية المستخلصة من التوزيعات المشار إليها مسبقا عند سرعة رياح 15 م/ث مع مسافة 10D بين التوربين والتوربين الذي يليه، وخلصت الدراسة إلي أن الانخفاض يعادل 20% لـ 3×3 (صفوف× أعمدة)، 32% لـ 4×4، 46% لـ 6×6، 12.8% لـ 16×1، 23.3% لـ 2×8 و 29% لـ 8×2 عندما تقارن هذه التوزيعات بالنسبة للتوزيع 1×16 كتوزيع أمثل من جانب الفقد في الطاقة المنتجة.

ABSTRACT

This study deals with the modeling of the wake effect on the energy extracted from the wind farms. It covers the wake effect of the interaction of the upstream wind rotor with/without the upstream right and/or upstream left wind rotor. A mathematical model representing a single wake model based on the linear description of the wake is developed in order to predict the wind speed inside the wake region at any downstream distance within the wind farm. Two different types of turbines with diameters of 62m and 100m are considered. Accordingly the effect of the wake on the energy produced from the wind farms is estimated.

A number of different wind farm layouts are studied. Case studies including 3×3, 4×4, 6×6, 1×16, 16×1, 2×8, and 8×2 layouts are considered. Extracted energy is calculated in each case and an optimum layout is determined from different layouts. The effectiveness of the other layouts with respect to the optimum is obtained. The results showed that there is a drop in the annual extracted energy from the above mentioned layouts depending on the W.T. distances separating the W.T's. The wind speed was assumed to be 15m/s with 10D downstream distances. The losses are estimated to be 20% for 3×3 (rows×column), 32% for 4×4, 46% for 6×6, 12.8% for 16×1, 23.3% for 2×8, and 29% for 8×2 when these layouts are compared to 1×16 layout as an optimum layout.

KEYWORDS: Wake effect; Wind farm; Downstream distance; Extracted energy; Wake wind speed; Wind speed behind the rotor.

INTRODUCTION

A wind farm contains a number of horizontal wind turbines, these W.T.s are positioned aligned in clusters facing the wind direction, they are eclectically connected together in one place depending on the available area of the project, and the quantity of energy required [2].

Each wind rotor generates a turbulent region called wake. This wake causes a sudden decrease in velocity, consequence it causes a decrease in the quantity of air and wind speed entering the downstream turbine, so that less energy will be produced by the downstream turbine.

As air comes out of the wind turbine rotor, its initial diameter is almost equals to the diameter of the turbine rotor. Then it tends to spread out conically.

The objective of this study is to estimate the effect of wake on energy extracted from a wind farm. This paper deals with a mathematical model for the effect of wake, which uses only the commonly available parameters and data of the wind farm and the turbines within the farm.

WAKE WIND SPEED (v_w)

The radius of the cone can be represented as a function of the downstream distance from the turbine location as follows;

$$\mathrm{r(x)} = \mathrm{r_{rot}} + \mathrm{x} * \tan\alpha \tag{1}$$

Where $\mathrm{r(x)}$ is radius of the shadow cone, r_{rot} is radius of upstream turbine, x is the distance between the turbines, and $\tan\alpha$ is the factor of the cone. The factor $\tan\alpha$ was found, to have two values; 0.04 for turbines facing the wind stream in the first row of the wind farm, and 0.08 for turbines downstream [3].

Corresponding to the following assumptions; steady and one dimensional incompressible flow with the basis of the principle of mass conservation, the wake wind speed can be computed at any downstream distance.

Referring to Figure (1) and to the integral continuity equation, at any location along the downstream distances, the mass flow rate is given by;

$$\frac{\partial m}{\partial t} = A(x) * v_w(x) * \rho \tag{2}$$

Also, at any location along the downstream distance, the sweeping wind speed has two different values, one inside the shadow cone (v_{w0}), and the other outside (v_0). This leads to the equation of the mass flow rate as;

$$\frac{\partial m}{\partial t} = A_{rot} * v_{w0} * \rho + \left(A(x) - A_{rot}\right) * v_0 * \rho \tag{3}$$

By equating equations (2) and (3), the wake speed at the downstream turbine location as a function of the free wind speed is given by the following equation;

$$v_w(x) = v_0 + \left(v_{w0} - v_0\right) * \left(\frac{r_{rot}}{r(x)}\right)^2 \tag{4}$$

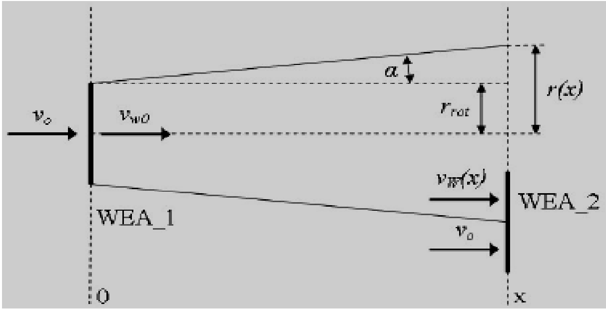

Figure 1: turbine and shadow cone [5]

Where $v_w(x)$ is the wake wind speed at distance x, v_0 is a free stream speed, v_{w0} is the wind speed behind the rotor, r_{rot} is the radius of the rotor, and $r(x)$ is the radius of the wake region a long distance x.

The calculation of wake effect from neighboring turbines is determined by assuming the square velocity deficit of a mixing wake to be equal to the sum of squared velocity deficit for each wake at the calculated downstream distance [4].

$$(v_0 - v)^2 = (v_0 - v_w)_1^2 + (v_0 - v_w)_2^2 + ... \tag{5}$$

The resultant wind speed over the rotor at distance x is:

$$v = v_0 - \sqrt{\sum_{for\ all\ wakes}^{n} (v_0 - v_w)^2} \tag{6}$$

POWER OUTPUT

The power associated with a mass flow rate entering the cross-sectional area of the rotor, that is a function of free wind speed is given by the equation;

$$P_0 = \frac{1}{2} * \frac{\partial m}{\partial t} * v_0^2 \tag{7}$$

The theoretical maximum extractable power is given by the equation;

$$P_{mech\_th} = \frac{1}{2} * \frac{\partial m}{\partial t} * \left(v_0^2 - v_{w0}^2\right) \tag{8}$$

The theoretical power coefficient is given as follows [1];

$$C_{P_{th}} = \frac{P_{th}}{P_0} = \frac{1}{2} * \left(1 + \frac{v_{w0}}{v_0}\right)\left(1 - \frac{v_{w0}^2}{v_0^2}\right) \tag{9}$$

WIND SPEED BEHIND THE ROTOR (v_{w0})

Referring to Betz limit, the physical possible amount of power extracted is not more than 59.3% of the power in the wind, and the minimum possible power is being zero**.** Thus it can be considered that the values of $C_{P_{th}}$ lie within the range of $0 \le C_{P_{th}} \le C_{P\_Betz}$. The important equation (9) which is the function of $C_{P_{th}}$ has different set of solutions. These can be represented as follows [5];

$$\frac{v_{w0}}{v_0} = \frac{4 * \cos(\varphi/3) - 1}{3} \qquad for\ C_{P\_th} < \frac{8}{27} \tag{10}$$

Or

$$\frac{v_{w0}}{v_0} = -\frac{4 * \cos(\theta/3) + 1}{3} \qquad for\ C_{P\_th} \ge \frac{8}{27} \tag{11}$$

With

$$\varphi = \cos^{-1}\left(1 - \frac{27}{8} * C_{P_{th}}\right)$$

And (12)

$$\theta = \cos^{-1}\left(\frac{27}{8} * C_{P_{th}} - 1\right)$$

By these equations (10) to (12), the relationship between the ratio of the wind speed behind the turbine to the speed in front of the turbine and the ideal power coefficient $C_{P_{th}}$ is established.

Normally, the turbine reaches its maximum efficiency at its design tip speed ratio thus,

$$\eta = \frac{C_{P_{max}}}{C_{P_{opt}}} \tag{13}$$

Where $C_{P_{opt}}$ represents the optimum power coefficient which can be considered as a measure for the approximation of the power losses. The optimum coefficient is only a function of the tip speed ratio and can be written as [5];

$$C_{P_{opt}} = \frac{16}{27} * \left(1 - \frac{0.219}{\lambda^2} - \frac{0.106}{\lambda^4} - \frac{2}{9} * \frac{\ln \lambda^2}{\lambda^2}\right) \tag{14}$$

Where λ is the tip speed ratio.

The theoretical power coefficient $C_{P_{th}}$ can be obtained from the following relationship;

$$C_{P_{tg}} = C_P * \left(C_{P_{opt}} / C_{P_{max}}\right) \tag{15}$$

Noting that $C_{P_{max}}$ can be also calculated from the actual power coefficient.

EXTRACTED ENERGY BY A TURBINE

The extracted energy from the turbine in a site can be calculated from the following equation;

$$E(v) = f(v) * P_e(v) \tag{16}$$

$P_e(v)$ is the power curve of the turbine, and $f(v)$ is the Weibull Distribution. This distribution can be calculated by using the following equation;

$$f(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} e^{-\left(\frac{v}{c}\right)^k} \tag{17}$$

Where k is the Weibull shape parameter, c is the Weibull scale parameter, and v is the wind speed. The scale parameter can be estimated from the following relation;

$$c = \frac{\overline{V}}{\Gamma\left(1 + \frac{1}{k}\right)} \tag{18}$$

Where $\overline{V}$ is the mean wind speeds.

The term $\Gamma\left(1 + \frac{1}{k}\right)$ can be calculated from the following approximate equation [6]:

$$\Gamma(X) = \left(\sqrt{2\pi X}\right)\left(X^{X-1}\right) e^{-X}\left(1 + \frac{1}{12X} + \frac{1}{288X^2} + \frac{139}{51840X^3}\right) \tag{19}$$

Where X is $\left(1+\frac{1}{k}\right)$

Integrating the above equation (16) over a year from cut-in wind speed to cut-out wind speed we get a simplified expression for energy extracted by turbine:

$$E_E = 8766 \int_{V_{Cut-in}}^{V_{Cut-out}} P_e(v) f(v) dv \tag{20}$$

Where E_E is the annual energy extracted by a turbine and 8760 is the number of hours per year.

CASE STUDIES

In this study, two different wind turbines are used, Type I has; three-blades and 2500 kW rated power with rotational speed of 14.45 r.p.m (revaluation per minute), and a rotor diameter of approximately 100m. The type II; three-blades and 1012 kW rated power with rotational speed of 18.9 r.p.m, and a rotor diameter of approximately 62m.

Three simplified layouts of wind farms are used: the first one is 3×3 layout in which 9 turbines are aligned in 3 columns by 3 rows. The second is 4×4 layout in which 16 turbines are aligned in 4 columns by 4 rows. The third is 6×6 layout in which 36 turbines aligned in 6 columns by 6 rows. Different downstream spacing distances are taken, 4D, 7D, and 10D where D is the turbine rotor diameter. A program was prepared to accomplish this task using all the formula explained above.

RESULTS AND DISCUSSIONS

The program uses the formulas discussed above in order to find the energy extracted from the wind farm.

VALIDATION OF THE CALCULATED WIND SPEED

For the validation of the results of the wind speed behind the rotor, that obtained using C_P model. The wind speed behind the rotor (v_{w0}) as a function of the free wind speed (v_0) was computed and compared with the results obtained using wake model that used in [3]. The later model uses the thrust coefficient (C_t) rather than the power coefficient which is used in this study. The discrepancy between the two results, are found minimal except for the practically unimportant range near and below the cut-in wind speed. Figure (2) illustrates the findings.

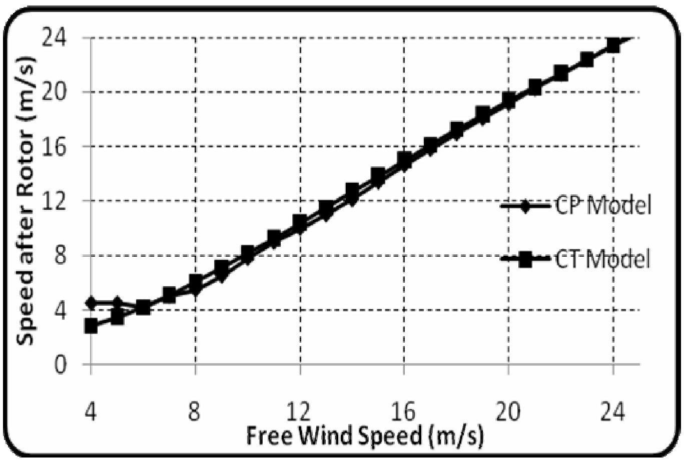

Figure 2: Comparison between C_P and C_t model

The developed program was run using the two selected turbines as case studies. The calculations are made in order to optimize the energy extracted from a specific layout of wind farm. The results obtained using the type I, and type II are illustrated in the next sections respectively.

RESULTS OF TURBINE 1

LAYOUT 3×3

The results of the annual extracted energy are shown in Figure (3), which illustrates the annual energy versus wind speed at different downstream distances.

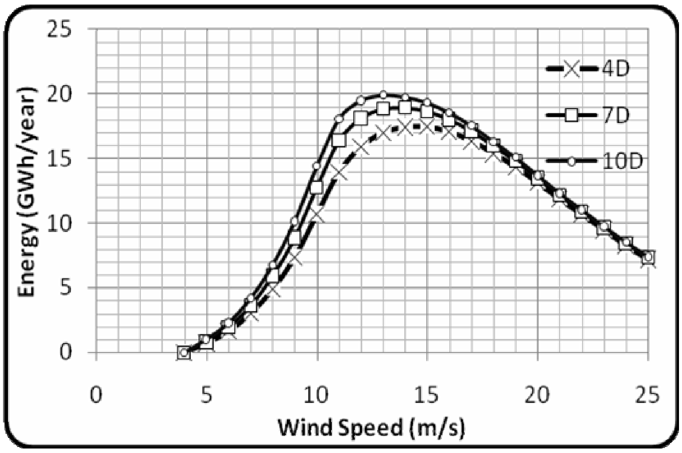

Figure 3: Annual energy extracted at different downstream distances

Form the above curve we can see that the maximum amount of energy per year can be extracted from 3×3 at almost 13 m/s wind speed by using 10D downstream distances.

LAYOUT 6×6

The results of using 6×6 layout are obtained and illustrated in Figure (4).

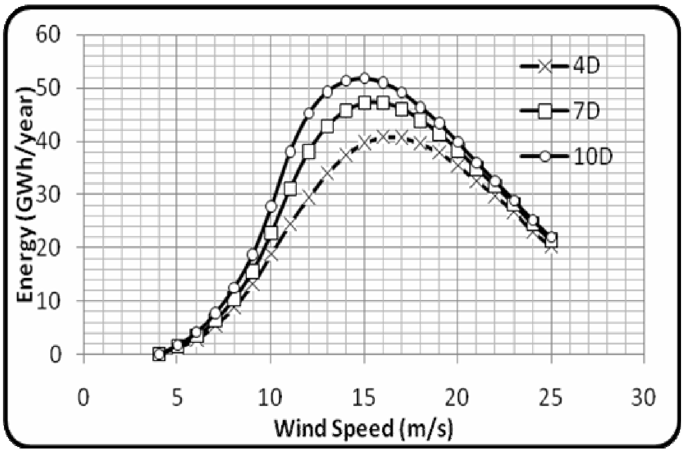

Figure 4: Annual energy extracted at different downstream distances

From the annual energy curve using 6×6 layout, it could be noticed that more energy can be extracted by increasing the number of turbines in the cluster; it reaches more than twice the amount of energy extracted using 3×3 layout at 10D downstream distance.

However, it could be concluded that the annual energy extracted per unit turbine is higher in 3×3 layout than that in 6×6 layout. This is due to the shadow effect, where the relative number of turbines in 6×6 layout that lie in the wake is more than those for 3×3 layout. For wind speed 15 m/s with 10D downstream distance, there is a decrease of energy of 32% when 6×6 layout is used compared to 3×3 layout.

RESULTS OF TURBINE 2

LAYOUT 3×3

The results of annual energy extracted from a cluster of 3×3 layout are obtained and illustrated in Figure (5).

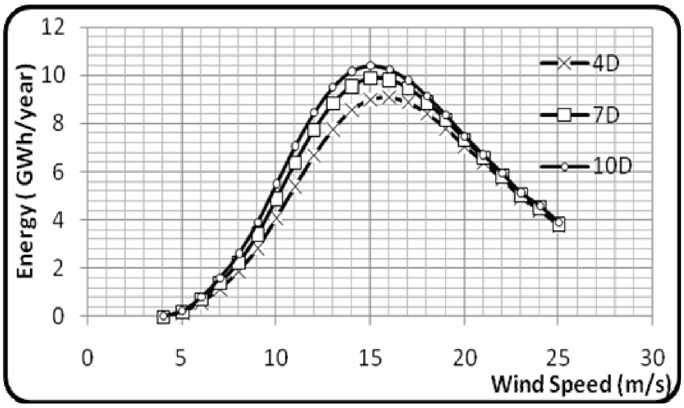

Figure 5: Energy curve at different downstream distances

Comparing curves of Figure (3) and (5), the maximum amount of energy is reduced to almost half of its value by reducing the diameter of rotor from 100m (Type I) to 62m (Type II) using the same layout 3×3.

LAYOUT 6×6

The results of the annual energy extracted are illustrated in Figure (6).

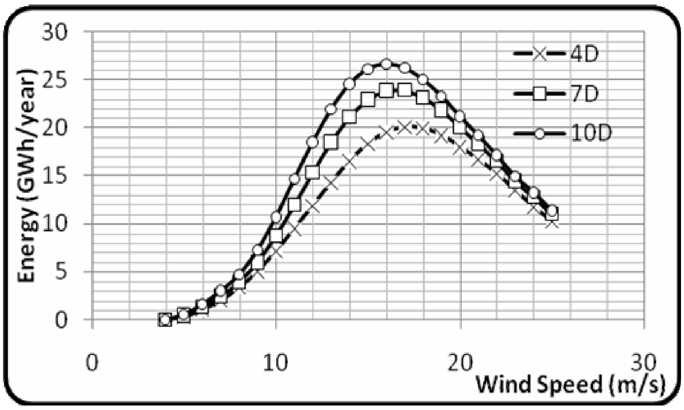

Figure 6: Energy curve for different downstream distances

Apparently, we can see that more energy can be extracted by increasing the number of turbines in the cluster. From all the results above we can conclude that increasing turbine diameter, number of the turbines in cluster, wind speed, and downstream distance, give more extracted energy from a wind farm.

OPTIMUM LAYOUT

The optimum energy from a wind farm depends on the turbines distribution in the cluster. Therefore, in this section we will consider different layouts for a specific wind farm. This farm consists of 16 turbines distributed in several configurations; 16 turbines aligned in one row upstream facing the wind. This represents the first case. The second case is to align the 16 turbines in one column downstream wind. The third case represents 16 turbines aligned in two rows each one has 8 turbines upstream wind. The forth case represents 16 turbines aligned in two columns. The last case represents 16 turbines aligned in 4 rows and 4 columns. All these cases are used in order to show which case is most affected by the wake.

The first case (1×16) could be considered as an optimum layout because the turbines are not affected by downstream distances and the turbines lie in cluster far away in between columns so that the effect of wake between the turbines in the same row is disappeared. Thus, wind farm layouts might be evaluated compared to this optimum farm layout, as percentage of each arrangement case by dividing its extracted energy with respect to the extracted energy of the 16 turbines in one row upstream wind (the first case). This percentage is presented in Figures (7), (8), and (9) versus the wind speed at different downstream distances.

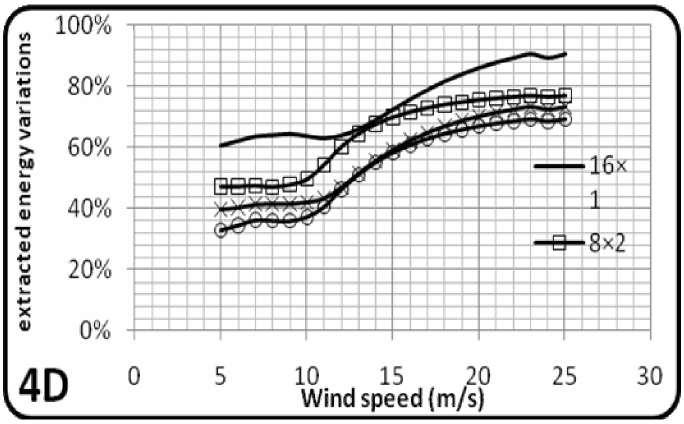

Figure 7: variation in extracted energy with respect (16×1) wind farm for 4D downstream distance

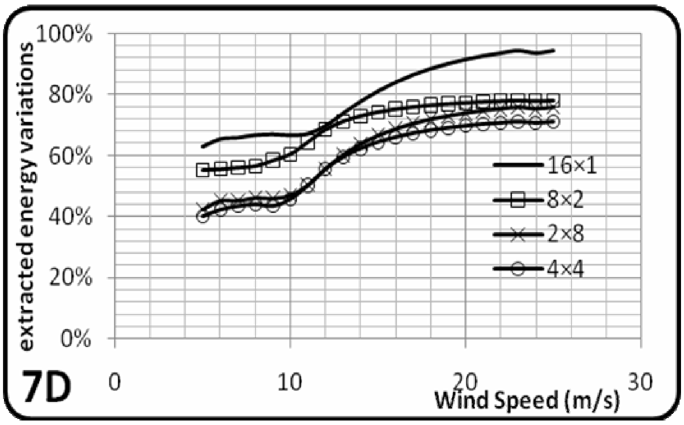

Figure 8: variation in extracted energy with respect (16×1) wind farm for 7D downstream distance

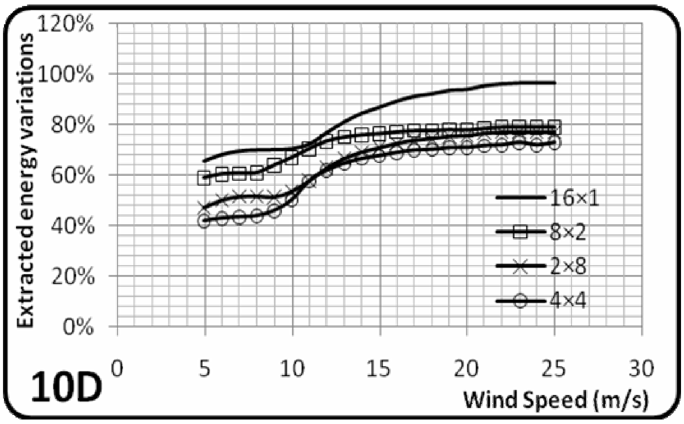

Figure 9: variation in extracted energy with respect (16×1) wind farm for 10D downstream distance

Apparently, anyone can see that distributing the turbines in one column gives more energy than distributing them in the other cases.

The loss of energy can be noticed from the above curves with respect to the first case. At wind speed about 14 m/s, the loss of maximum energy could be extracted from the above cases were; 15%, 25%, 30%, and 35% from the optimum energy using the second (16×1 layout), the third case (2×8), the forth case (8×2) and the last case (4×4) respectively.

CONCLUSIONS

Referring to the mathematical model and various results that are presented, there are a number of points that may be considered in this study. These are as follows;

- The annual extracted energy, for all case studies, increases with the wind speed in the range between cut-in wind speed and rated wind speed. This is the actual range of the wind speed that occurs due to the natural wind.
- The energy extracted from wind farms increases both with increasing downstream distances, which will reduce the wake effect, and with increasing the rotor diameter, so that the maximum annual energies extracted using 6×6 layout from type I, which has 100 m diameter, are 39 GWh/year, 47 GWh/year, and 52 GWh/year with 4D, 7D, 10D downstream distances, respectively. Whereas the maximum annual energies extracted from type II, which has 62 m diameter, are 20 kWh/year, 24 kWh/year, and 27 kWh/year with 4D, 7D, and 10D downstream distances respectively.
- Different distributions are used for 16 turbines and an optimum layout is determined, which includes all the turbines aligned upstream the wind. It has been found that the extracted energy is lower for different distribution of the same number of turbines. This is estimated to be 15% for one column downstream (16×1), 25% for two columns upstream (2×8), 30 % for two columns downstream (8×2), and 35% for squared grid (4×4) for wind speed 14 m/s with 10D downstream distance.
- The square layout wind farm is not recommended due to the drop in the extracted energy that could reach 35% due to the wake effect. This means installing wind turbines inside the wind farm surrounded by other turbines at the boundary should be avoided due to the higher effect of the wake on the performance of the wind farm.

Recommendations

It might be useful as future work to take the following recommendations into consideration:

- Visual basic program, which has been used in this study, could be extended to include the other models.
- The effect of atmospheric stability on power production and the effect of hub height are recommended for further studies.
- The program could be extended to include the effect of wake in between the turbines in one row.
- The program could be extended to include the staggered arrangement.

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