ANALYSIS OF SKEWED BRIDGES BY THE COUPLING OF FINITE ELEMENT AND BOUNDARY ELEMENT METHODS

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الملخص

تتناول هذه الورقة تحليل الجسور المنحرفة وذلك باستخدام تقنية الدمج بين طريقة العناصر المتناهية وطريقة العناصر الحدودية. الشكل الهندسي للجسر المنحرف تم تشكيله باستخدام عناصر حدودية من الدرجة الخطية والدرجة الثانية. عند ركن الجسر اخذ في الاعتبار تأثير الالتواء في صياغة العناصر الحدودية. ترتبط العناصر المتناهية والعناصر الحدودية بواسطة عدد من العقد البينية في الاتجاه الطولي. معادلات العناصر المتناهية تم تحويلها إلى معادلات عناصر حدودية وذلك بعد تحقيق آلية التوافقية عند السطح البيني بين الطريقتين.

تم تحليل نوعين من الجسور المنحرفة وذلك باستخدام الطريقة الموحدة لتوضيح حسن أداء وفوائد هذه الطريقة مقارنة بالطرق العددية الأخرى في تحليل الجسور. النتائج المتحصل عليها باستخدام كلا من طريقة العناصر المتناهية والطريقة الموحدة وجدت متقاربة، حيث وجد أن أقصى فرق في قيم الانحراف والعزوم لا يتعدى نسبة 4%.السهولة والتقليص في كمية البيانات الداخلة موضحان في المثال الثاني، حيث تم تقليص البيانات الداخلة بين الطريقتين بنسبة تفوق 80%.

ABSTRACT

In this paper, the analysis of skewed bridges using the coupling technique of finite element and boundary element methods is presented. The geometry of the skewed bridge is modeled using linear and quadratic boundary elements. At the corner of the bridge deck, the torsional effect is considered in the boundary element formulation. The finite elements and boundary elements are connected at a number of interface nodes in the longitudinal direction. The finite element equations are transformed into boundary element equations and the compatibility interface mechanism required to combine the two methods is developed.

Two skewed bridges are analyzed using the combined method to illustrate the performance and the advantages of the combined method over the other numerical techniques. The results obtained using boundary element and finite element methods are in a good agreement, where the maximum difference in deflection and moment results is about 4%. The simplicity and reduction in input data are illustrated in example two, where the input data is reduced by more than 80%.

KEYWORDS: Combined Method; Finite Element; Boundary element; Bridges.

1. INTRODUCTION

One of the most interesting features of the Boundary Element Method (BEM) is that it is easy to combine the technique with the other numerical methods such as Finite Element Method (FEM). In order to profit from the advantages of the two numerical techniques (e.g. BEM and FEM), a combination between them seems ideal. Such a combination should allow for the use of the most appropriate technique over each domain of a problem with a reduced number of operations and without compromise in Accuracy. In many problems, the boundary element method may provide the appropriate conditions to represent a large or infinite domain while the finite element method can solve complex material properties in a finite domain.

The coupling techniques of the boundary element and finite element methods have been studied by many researchers. Mainly two different approaches were used for the coupling technique. The first approach consists of transforming the boundary element equations into a stiffness system where a large number of operations are required to achieve the stiffness matrix. In the second approach the finite element equations are directly combined with the boundary element equations, forming a square system of equations which includes the interface tractions as unknowns plus the unknowns associated with the boundary element region. The coupling was first discussed by Zienkiewicz et al [1]. Energy functional was used in combination with the boundary integral equations which were derived from the collocation method.

Brebbia and Georgiou [2] examined the coupling of BEM and FEM for two dimensional elastostatics problems using the two different approaches. A few numerical examples were considered in order to examine the combined solution for two-dimensional elastostatics problems. A program was developed by the authors who combined constant boundary elements with quadratic finite elements and although the coupling techniques were not fully compatible, it still gave good results in practice.

Kelly et al. [3] summarized different procedures for combining the boundary element method with the other numerical techniques. He applied the boundary element method to solve potential problems. Non-symmetric and symmetric stiffness formulations were compared using quadratic shape functions. In one example the symmetric matrix gave better results than the non-symmetric one.

Hung and Dawkins [4] analyzed the behaviour of a U-frame structure using the coupling technique. Finite elements were used to simulate the U-frame structure while the surrounding soil mass was modeled using boundary elements. Isoparametric quadrilateral finite elements and linear boundary elements were used. The authors assumed that the horizontal displacements of the soil were negligible at a sufficient distance from the structure centerline while the vertical displacements were negligible at a certain depth below the ground surface.

In this paper, the second approach is adopted and the compatibility requirements at the interface between the boundary element and finite element regions are satisfied.

The BEM is capabable to model the geometry of the skewed bridges using one dimensional linear and high order elements.

At the corner of the bridge, the effective corner force due to the difference in the twisting moments is considered in the formulation of the boundary integral equations. Along the interface, double nodes at the corner were used. Both nodes have the same coordinates, but may have different boundary conditions.

The finite element equations were used to model the other bridge components such as girders, diaphragms, etc… Two numerical examples of skewed bridges are presented to show the validity and accuracy of the combined method.

2. DESCRIPTION OF THE COUPLING THEORY

In this paper, the solution will be developed for a skewed slab-on-girder bridge. Consider a problem of a slab on two girders consisting of two domains, R1 and R2, joined by interfaces I_1 and I_2 as shown in Figure (1). The domain R1, which is the slab deck, is modeled by the boundary elements, while the finite elements are used to model the two girders.

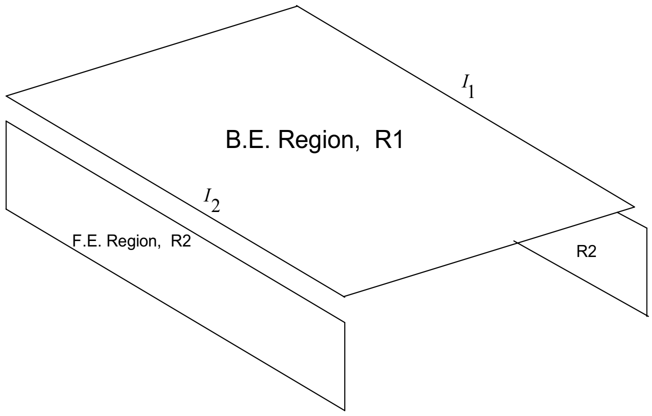


Figure 1: Boundary element and finite element regions

The boundary element equations for the slab deck [1] can be written as:

$$\left[(H_b)_I \;\; (H_b) \right] \begin{Bmatrix} (D_b)_I \\ (D_b) \end{Bmatrix} = \left[(G_b)_I \quad (G_b) \right] \begin{Bmatrix} (P_b)_I \\ (P_b) \end{Bmatrix} + \{q\} \tag{1}$$

Where I represents the boundary interface between BE and FE regions, and b represents the boundary element region. [H] and [G] are matrices include the coefficients corresponding to the boundary displacements and forces respectively, {q} is the domain integral, the vectors {D} and {P} can be written as:

$$D = \begin{Bmatrix} U \\ V \\ W \\ \theta_n \end{Bmatrix} \quad \text{and} \quad P = \begin{Bmatrix} N_x \\ N_y \\ S \\ M_n \end{Bmatrix} \tag{2}$$

U, V and W are the in plane and vertical boundary deflections, and θ_n is the normal slope respectively. Nx, Ny, S and M_n are the in plane boundary forces, effective corner force and normal moment respectively, see Figure (2).

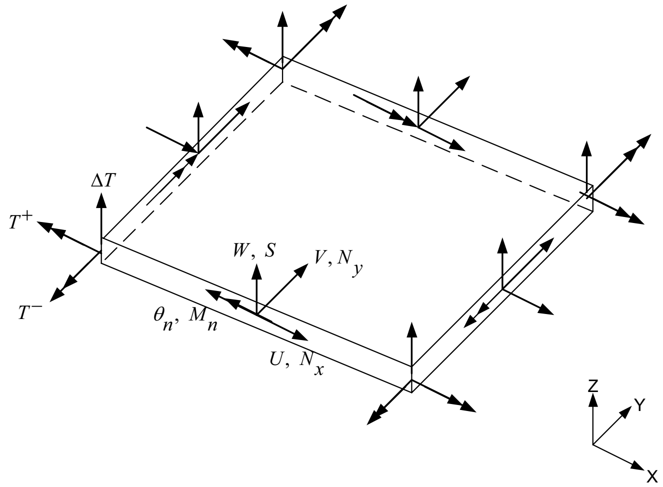


Figure 2: Direction of displacements and forces along the boundaries

Similarly, the finite element equations for both girders can be written as:

$$\begin{bmatrix} K_{g1} & K_{g2} \end{bmatrix} \begin{Bmatrix} D_{g1} \\ D_{g2} \end{Bmatrix} = \begin{Bmatrix} F_{g1} \\ F_{g2} \end{Bmatrix} \quad (3)$$

Where [K], {F} and {D} are the stiffness matrix, nodal force and nodal displacement vectors respectively, and g_1 and g_2 denote the two girders.
Equation (3) can be simplified by condensing the non-interface degrees of freedom to include only the interface degrees of freedom using the condensation technique described in [1]. This process is used to reduce the size of the global stiffness matrix, therefore, equation (3) becomes;

$$[\ (K_g)_I\ ]\ \{\ (D_g)_I\ \} = \{\ (F_g)_I\ \} \quad (4)$$

In order to combine equations (1) and (4), the nodal forces in the finite element equation must be written in the form of traction as follow:

$$\{F\} = [T]\ \{P\} \quad (5)$$

Where [*T*] is a distribution matrix that transforms the tractions *P* into equivalent nodal forces as follow:

$$[T] = \int_{-1}^{+1} \{L_1\}^T \{L_2\}\ |\ J\ |\ d\xi| \quad (6)$$

Where L_1 and L_2 are interpolation functions for displacements and forces and *J* is the Jacobian. Therefore, equation (4) can be written as:

$$[\ (K_g)_I\ ]\ \{\ (D_g)_I\ \} = [T]\ \{\ (P_g)_I\ \} \quad (7)$$

The compatibility of displacements and forces equilibrium conditions at the interface require that the displacements at the interface I between the slab deck and girders must be equal and the sum of forces must be equal to zero, i.e.

$$\begin{aligned} &\{D_b\}_I = \{D_g\}_I = \{D_I\} \\ &\{P_b\}_I + \{P_g\}_I = 0. \qquad or \\ &\{P_b\}_I = -\{P_g\}_I = \{P_I\} \end{aligned} \tag{8}$$

Substitute the conditions of equation (8) into equations (1) and (7), then both equations can be combined together to form a single matrix expression as:

$$\begin{bmatrix} (H_b)_I & H_b & -(G_b)_I \\ (K_g)_I & 0 & T \end{bmatrix} \begin{Bmatrix} D_I \\ D_b \\ P_I \end{Bmatrix} = [G_b]\ \{P_b\} + \{q\} \tag{9}$$

Equation (9) represents the hybrid solution and can be solved for all boundary unknowns after imposing the boundary conditions.

3. NUMERICAL EXAMPLES

Two skewed bridges are tested to verify the performance of the hybrid solution and to show the advantages of the coupling technique. The results obtained from the combined method are compared with the finite element and finite strip solutions.

Example 1: A concrete skew slab on two girders with an angle of 10 degree is analyzed. The loading consists of a point load of 110 kN distributed uniformly on a small patch [5]. The dimensions and material properties of the bridges are given in Figure (3). Two different boundary meshes are used to test the performance of the combined method. The boundary element and finite element idealizations are shown in Figure (4).
The results of the deflections and moments for the two meshes are compared with the finite strip solution and summarized in Tables (1) and (2). The results from the two solutions are in good agreement.

Example 2: A simply supported skew concrete slab bridge under two trucks is analyzed to show the advantages of using BEM over FEM in terms of simplicity and reduction of input data. The dimensions, material properties and the idealization of the bridge are shown in Figure (5). In the finite element analysis, the bridge is discretized into 84 rectangular elements with a total of 291 nodes, while the boundary element mesh consisted only of 16 quadratic elements with 36 nodes.
The results of the longitudinal moments and central deflections are given in Table (3).
As we can see, the BEM is more efficient than FEM when the bridges are subjected to moving loads. The position and number of loads do not change the boundary element mesh where only the boundaries need to be discretized, while the finite element mesh needs to be changed as the loads are moving over the bridge. In addition to that, the FE mesh requires more refinement near the leads, thus it leads to huge number of simultaneous equations and large band width.

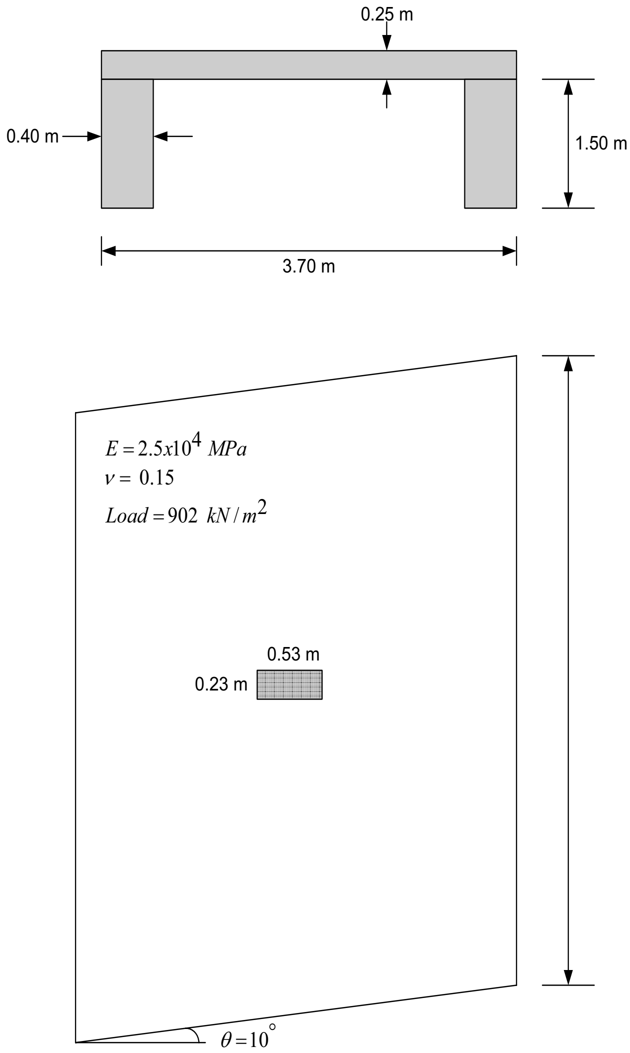
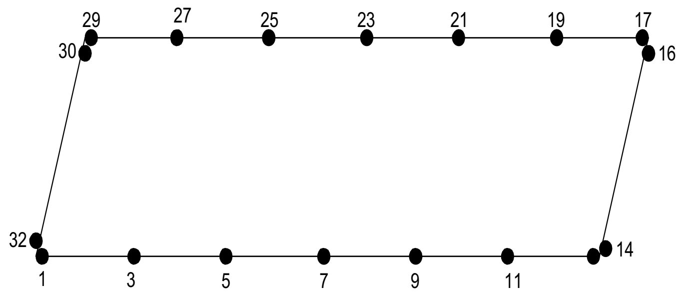
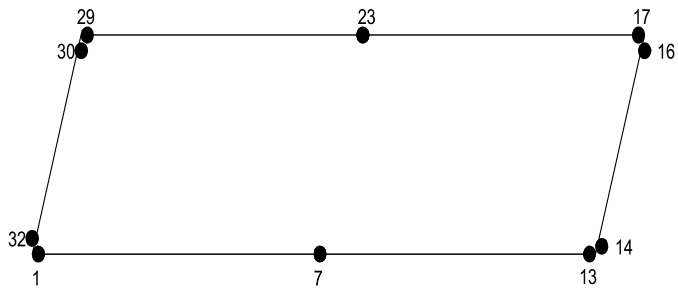


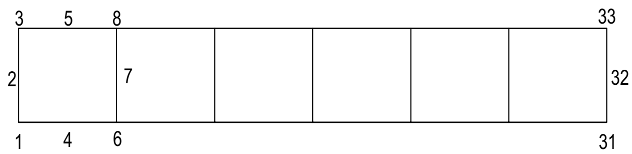
Figure 3: Dimensions and materials properties for example 1



Mesh 1 for the bridge deck

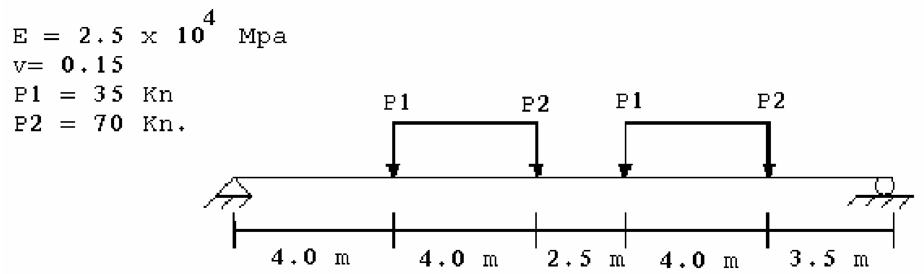


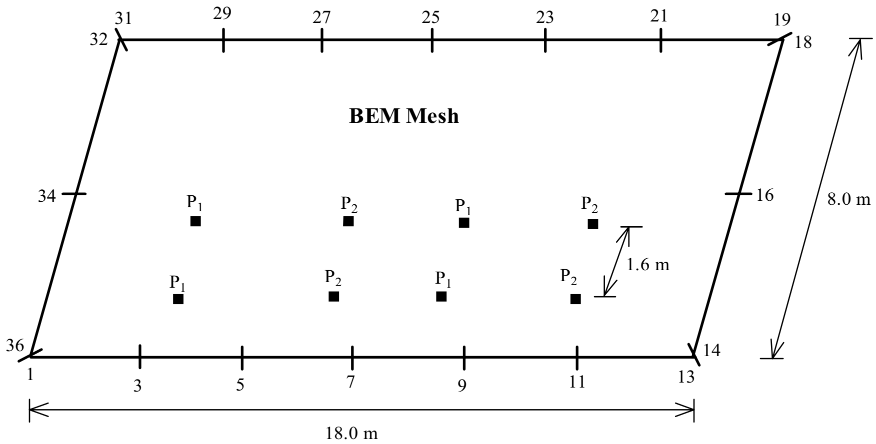
Mesh 2 for the bridge deck



6 X 1 Mesh for the girders

Figure 4: BE and FE idealizations for example 1





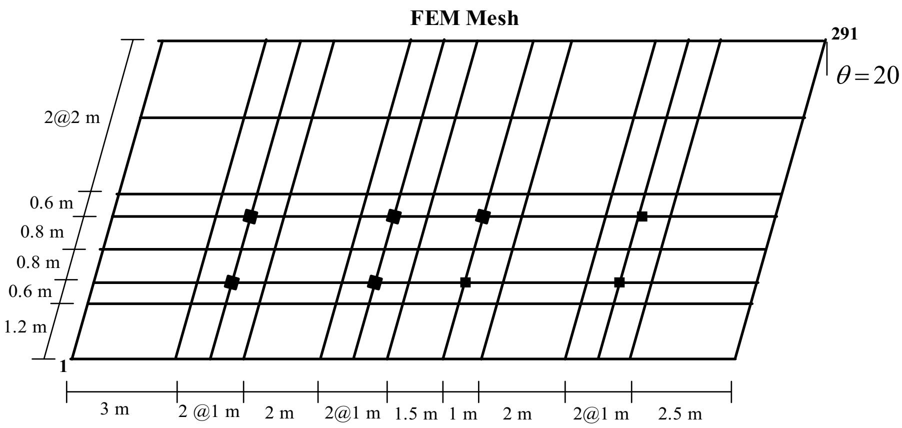


Figure 5: Details of the bridge in example 2

Table 1: Deflection along the span

Span (m)	FSM (kN.m/m)	BEM-FEM (kN.m/m)	
		Mesh 1	Mesh 2
1.0	0.105	0.109	0.106
2.0	0.237	0.246	0.242
2.5	0.322	0.333	0.328
3.0	0.421	0.433	0.428
3.5	0.521	0.534	0.529
3.885	0.574	0.568	0.563
4.0	0.580	0.589	0.587

Table 2: Longitudinal moment along the span

Span (m)	FSM (kN.m/m)	BEM-FEM (kN.m/m)	
		Mesh 1	Mesh 2
1.0	1.60	1.65	1.71
2.0	4.47	4.56	4.54
2.5	6.97	7.06	7.06
3.0	10.80	10.83	10.82
3.5	17.23	17.03	17.01
3.885	25.56	24.78	24.74
4.0	26.82	25.99	25.94

Table 3: Vertical Deflection and Longitudinal Moment along the span

Span (m)	BEM		FEM	
	W(mm)	Mx (kN/m)	W(mm)	Mx (kN/m)
1.5	38.3	3.83	38.1	5.62
3.5	85.29	10.19	84.78	13.34
4.5	105.3	12.22	104.7	16.38
6.0	129.5	13.60	128.7	19.77
7.5	145.2	19.76	144.3	23.72
8.5	150.2	20.78	149.2	24.72
9.75	149.8	18.78	148.9	23.92
11.0	142.4	17.07	141.5	22.64
14.0	98.8	16.69	98.0	17.50
15.0	77.2	14.78	76.6	15.76
16.75	33.5	4.87	33.2	5.80

4. CONCLUSION

The hybrid solution can be effectively applied to analyze skew bridges. The boundary element is capable to model the boundaries of the skewed bridge. Using high order elements in the boundary element idealization is not necessary to obtain accurate results. However, high order elements reduce the required input data. In general, the examples demonstrate the validity and accuracy of the combined method.

5. REFERENCES

[1] Zienkiewicz, O.C., Kelly, D.W. and Bettess, P. (1977), The coupling of finite element and boundary element solution procedure, Int. J. Num. Math. Eng., Vol 11, 355 – 376.

[2] Brebbia, C.A. and Georgiou, P. (1979), combination of boundary and finite elements in elastostatics. Appl. Math. Modeling, Vol. 3, 212-220.

[3] Kelly, D.W., Mustoe, G.G. and Zienkiewicz, O.C. (1979), coupling boundary element methods with other numerical methods, Developments in boundary element methods-1, 251-258, Applied Science Publishers.

[4] Hung, C. and Dawkins, W. P. (1980), Stress analysis of a U-frame structure by the coupling of boundary and finite element methods. Engineering analysis, Vol. 5 No.2, 58-63.

[5] Galuta, E.M., Combined boundary element and finite element analysis of composite bridges, Ph.D. Thesis, Dept. of civil Eng., University of Ottawa, 1993.