MODELLING OF THE NON-LINEARITY OF STRESS-STRAIN CURVES FOR COMPOSITE LAMINATES

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الملخص

يقدم هذا البحث نموذج رياضي مبني على معادلات رياضية ونظريات ميكانيكا المواد المركبة وذلك لدراسة عدم إستقامة مخططات الإجهاد والإنفعال وإستنباط مسبباتها للمواد المركبة البلاستيكية المعززة بالألياف الزجاجية. النموذج الرياضي تم تشغيله بإستخدام برنامج حاسوب صمم لهذا الغرض باستخدام لغة Visual Basic وقد أختبرت عدة عوامل في البرنامج كمسببات لعدم استقامة المخططات منها التشققات الداخلية الدقيقة التي تحدث في المادة ودوران الألياف وإنفصالها عن المادة اللاصقة اثناء التحميل وكذلك التلدن والتشكل الذي يحدث للمادة البلاستيكية بسبب الرطوبة ودرجات الحرارة العالية التي تتعرض لها المادة المركبة اثناء الخدمة. تم تشغيل البرنامج والحصول على نتائج جيدة لمخططات الإجهاد والإنفعال. وتمت مقارنتها بمخططات عملية من دراسات سابقة حيث أستنتج من هذة الدراسة أن التشققات الداخلية الدقيقة ودوران الألياف اثناء التحميل وتلدن المادة البلاستيكية بسبب الرطوبة والحرارة هي من العوامل الرئيسية المسببة لعدم إستقامة مخططات الإجهاد والإنفعال في المواد المركبة.

ABSTRACT

This paper presents a constitutive model to predict the nonlinearity of the composite laminates when they are subjected to uniaxial or biaxial loading. The model is based on micromechanics equations and classical lamination theory. The constitutive model was implemented in a Visual Basic Computer Program. Matrix cracking, fibre rotation and plasticization of the resin are chosen as nonlinear factors in the model. Different experimental results were chosen from the literature to test the model. The modelled stress-strain curves showed substantial nonlinearity up to failure. Good agreement between the experiential and modelled results was achieved in most cases.

KEYWORDS: Composite Materials; Matrix cracking; Fibre rotation; resin; Plasticization; stress-strain curve; non-linear behaviour.

INTRODUCTION

Composite materials (or composites for short) are engineered materials made from two or more constituent materials that remain separate and distinct on a macroscopic level while forming a single component. There are two categories of constituent materials: matrix and reinforcement. At least one portion (fraction) of each type is required. The matrix material surrounds and supports the reinforcement materials by maintaining their relative positions. The reinforcements impart special physical (mechanical and electrical) properties to enhance the matrix properties. A synergism produces material properties unavailable from naturally occurring materials. Due to the wide variety of matrix and reinforcement materials available, the design potential is incredible [1].

Many commercially produced composites use a polymer matrix material often called a resin such as polyester, vinyl ester, epoxy, phenol, polyimide, polyamide, and others. The reinforcement materials are often fibers such as glass and carbon [2].

Applications of composite materials are now confound on either the hi-tech applications such as aerospace or military which utilized high grades of composites with less concentration on the cost, or the low grades of application where there are less design constraint. However, the future of composite materials is promising and we will not be exaggerating if we say that the future is for composite materials to substitute many of the other traditional materials such as steel and aluminum etc. Figure (1) shows some of the new models of industrial application which utilized composite materials [3].

(a) (b) (c)

Figure 1 : The industrial potential of composite materials (a): Car manufacturing, (b): Marine industry and (c): Oil industry.

However, until recently most of the previous design work was made on the experimental basis and with less contribution of the computer programming skills. With the tremendous computer development at present, the need for material behaviour modelling is demanding. Many efforts on this issue have been evolved [1]. For the complicated composite material many models for the linear elasticity are now in use by various computer packages. However, the non-linear behaviour modelling is still an active area of research. Therefore, this work is considered which deals with modelling the non-linear behaviour with different tools which is the lamination theory with micro-mechanics [4].

It has been known that the stress-strain relationship of the reinforced composite materials is initially linear for lower strain and then becomes totally non-linear up to failure. The non-linearity is significantly contributed by damages such as a matrix cracking, fibre rotation and yielding of the resin [5]. The effect of matrix cracking on the performance of the properties of composite structures is a degradation of stiffness due to the redistribution of stresses and variations of strain in cracked laminate resulting in non-linearity of stress-strain behaviour. It has been found that the overall average strain of cracked laminate is higher than the uniform strain in uncracked laminate. This is in addition to the average strain underlying stiffness reduction [6,7]. In recent years, various models have been developed to evaluate the loss of stiffness in cracked laminates [8-10].

The non-linear behaviour of stress-strain for reinforced-polymer composite materials has been studied extensively in the literature. Petit and Waddoups [4] presented an analytical technique to predict the non-linear behaviour for a laminated composite. The model was based on the lamina theory and incrementally average laminate stresses. Sun and Tao [11] modelled the stress-strain behaviour of composite laminate by considering both material non-linearity and progressive matrix cracking. Gibson and Fahrer [12] developed an empirical equation to account the effect of plasticization of the resin on the stress-strain behaviour. Fibre rotation during the loading of the composite laminate can contribute the nonlinearity of stress-strain curves. Gibson et al [13] presented a geometric equation to predict the incremental change of the angle of the fibre by updating the fibre angle for each increment of the load in term of hoop and longitudinal strains. This equation was adapted to account for the nonlinear parameter in this work.

THEORTICAL ANALYSIS

For the purpose of modelling it is assumed that the an angle ply laminate with an even number of plies at $\pm\theta$ to the axial direction subjected by biaxial load as shown in Figure (2).

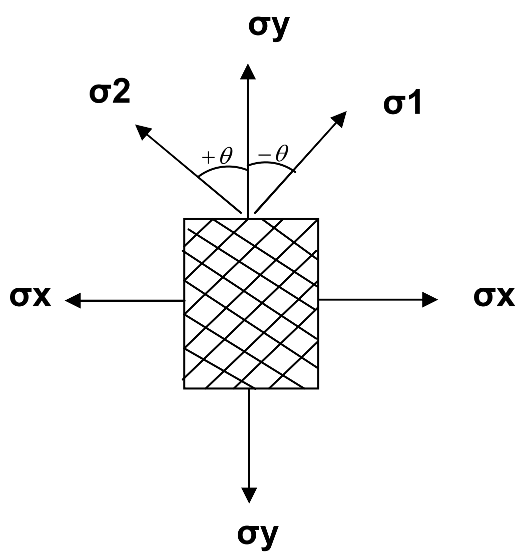

Figure 2: coordinate systems for composite laminate.

The elastic constants were related to those of the fibre and matrix by the Halpin-Tsai [2,3] equations:

$$E_{11} = E_f V_f + E_m V_m \quad ; \quad E_{22} = E_m \frac{1+\xi_{22}\eta_{22}V_f}{1-\eta_{22}V_f} \quad ; \quad G_{12} = G_m \frac{1+\xi_{12}\eta_{12}V_f}{1-\eta_{12}V_f} \qquad (1)$$

$$\nu_{12} = \nu_f V_f + \nu_m V_m$$

Where:

$$\eta_{22} = \frac{(E_f / E_m)}{(E_f / E_m) + \xi_{22}} \quad \text{and} \quad \eta_{12} = \frac{(G_f / G_m) - 1}{(G_f / G_m) + \xi_{12}} \tag{2}$$

Obviously, the parameters ξ_{11} and ξ_{12} take different values, which may be deduce or fitted when modelling a particular property. Hull [2] demonstrated that these parameters often take values 0.2 and 1 for ξ_{11} and ξ_{12} respectively The reduced stiffness constants of unidirectional ply were calculated [2] by:

$$Q_{11} = \frac{E_{11}}{1 - \nu_{12}\nu_{21}} \;; \qquad Q_{22} = \frac{E_{22}}{1 - \nu_{12}\nu_{21}} \;, \qquad Q_{12} = \nu_{12} Q_{22} \;; Q_{66} = G_{12} \tag{3}$$

The above reduced stiffness constants were transformed to obtain the corresponding values in coordinate system of the laminate, equivalent to a rotation of (θ). Therefore, the transformed reduced stiffness constants were obtained [3] by

$$\overline{Q} = [R] Q_{ij} \tag{4}$$

Where:

$$[R] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\cos\theta\sin\theta \\ \sin^2\theta & \cos^2\theta & -2\cos\theta\sin\theta \\ -\cos\theta\,\sin\theta & \cos\theta\,\sin\theta & \cos^2\theta - \sin^2\theta \end{bmatrix} \tag{5}$$

This lead to:

$$\begin{aligned} \overline{Q}_{11} &= Q_{11} c^4 + 2(Q_{12} + 2Q_{66}) s^2 c^2 + Q_{22} s^4 \\ \overline{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66}) s^2 c^2 + Q_{12} (s^4 + c^4) \\ \overline{Q}_{22} &= Q_{22} c^4 + 2(Q_{12} + 2Q_{66}) s^2 c^2 + Q_{11} s^4 \\ \overline{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}) s^2 c^2 + Q_{66} (s^4 + c^4) \end{aligned} \tag{6}$$

The elastic constants of the laminate were evaluated as follows:

$$\begin{aligned} E_y &= \overline{Q}_{11} (1 - \nu_{yx} \nu_{xy}) \\ E_x &= \overline{Q}_{22} (1 - \nu_{yx} \nu_{xy}) \\ G_{xy} &= \overline{Q}_{66} \\ \nu_{yx} &= \frac{\overline{Q}_{12}}{\overline{Q}_{22}} \\ \nu_{xy} &= \frac{\overline{Q}_{12}}{\overline{Q}_{11}} \end{aligned} \tag{7}$$

The biaxial stresses can be evaluated by pressurizing a thin walled filament wound composite pipe with internal pressure [2]. These stresses are:

$$\sigma_x = \frac{Pd_i}{2t} \quad and \quad \sigma_y = \frac{Pd_i}{4t} \tag{8}$$

The corresponding strains in coordinate systems can be calculated as follows;

$$\varepsilon_x = \frac{1}{E_x}\left(\sigma_x - \nu_{yx}\sigma_y\right) \quad ; \quad \varepsilon_y = \frac{1}{E_y}(\sigma_y - \nu_{xy}\sigma_x) \tag{9}$$

For uniaxial tensile loading ($\sigma_x = 0$), the tensile stress is given by:

$$\sigma_y = \frac{F}{A} \tag{10}$$

The corresponding strains become:

$$\varepsilon_x = \frac{1}{E_x}\left(-\nu_{yx}\sigma_y\right) \quad ; \quad \varepsilon_y = \frac{1}{E_y}(\sigma_y) \tag{11}$$

These strains were transformed using equation (5) to find the strains of unidirectional ply [12] as:

$$\begin{aligned} \varepsilon_1 &= \varepsilon_y \cos^2\theta + \varepsilon_x \sin^2\theta \,; \\ \varepsilon_2 &= \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta ; \\ \varepsilon_{12} &= 2(\varepsilon_x - \varepsilon_y)\sin\theta\cos\theta \end{aligned} \tag{12}$$

The stresses of the unidirectional ply [17] are:

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{pmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_{12} \end{pmatrix} \tag{13}$$

The above analysis enables the low strain elastic behaviour of the angle ply laminate to be modelled. At higher strain, however, there is significant non-linearity. These were assumed to be due to combination effects of matrix cracking, fibre rotation and resin plasticity. The degradation of the transverse and the shear due to matrix cracking can be estimated by [11]:

$$E_2 = E_2^o \exp(z_2\rho) \quad ; \quad G_2 = G_2^o \exp(z_3\rho) \tag{14}$$

ρ is a crack density function, which can be estimated [14] by:

$$\rho = k\left[\frac{\sigma_2 - \sigma_{fm}}{\sigma_{fm}}\right]^{\frac{1}{2}} \tag{15}$$

Where: σ_2 is the transverse stress of unidirectional lamina, σ_{fm} is the failure strength of the matrix.

$$\kappa = \sqrt{\frac{(E_1 + E_2)G_{12}}{E_1 E_2}} \tag{16}$$

In order to allow for the effect of rotation of the fibres on non linearity of the stress- strain curves, the value of the reinforcement angle is updated for each increment of pressure [15] by:

$$\theta = \arctan[(\frac{1+\varepsilon_x}{1+\varepsilon_y}) * \tan\theta_o] \quad (17)$$

The complex strain in the matrix is characterised by an equivalent strain [16]:

$$\bar{\varepsilon} = \sqrt{K_1(\varepsilon_1)^2 + K_2(\varepsilon_2)^2 + K_{12}(\varepsilon_{12})^2} \quad (18)$$

Where K_1, K_2 and K_{12} are the stress concentration factors for longitudinal, transverse and shear directions respectively. The K_1 is equal to unity because the strain in fibre direction (ε_1) is constant due to uniform displacement. However, in the transverse and shear directions, the strains are not uniformly distributed due to the effect of strain concentration in matrix around the fibre [16]. These concentration factors can be calculated [17] by:

$$K_2 = \frac{\frac{E_m}{E_2}}{1 - \frac{d}{l}(1 - \frac{E_m^{\ o}}{E_f})} \quad (19)$$

$$K_{12} = \frac{\frac{G_m}{G_{12}}}{1 - \frac{d}{l}(1 - \frac{G_m}{G_f})} \quad (20)$$

Where $E^o{}_m = \frac{E_m}{1 - {v_m}^2}$ (21)

Where *d* is the diameter of fibre and *l* is the closest centre-to-centre spacing of fibres. It is assumed that the fibres can be considered to be arranged on a hexagonal lattice [17] as shown in the Figure (3).

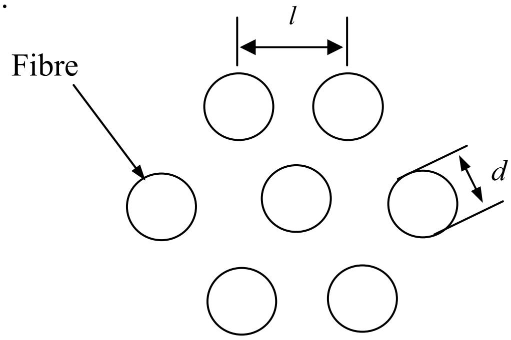

Figure 3: Hexagonal packing of unidirectional fibres [17].

If the matrix undergoes plastic deformation, the secant Young's modulus can be expressed in term of equivalent strain [18] as:

$$E_m = \frac{E_o}{k\bar{\varepsilon}}[1 - \exp(-k\bar{\varepsilon})] \quad (22)$$

MODELLING PROCEDURE

Figure (4) shows the flowchart of the iteration procedure which is followed in the Model development for predicting the elastic constants and stress-strain values for $\pm\theta$ angle ply laminate under uniaxial or biaxial loading. First, the model started with initial conditions which vary according to the mechanical properties of laminate need to be modelled (see Table 1) then it calculates the elastic constants, reduced stiffness, transformed reduced stiffness and stresses and strains in coordinate and principle systems using equations (1-13).

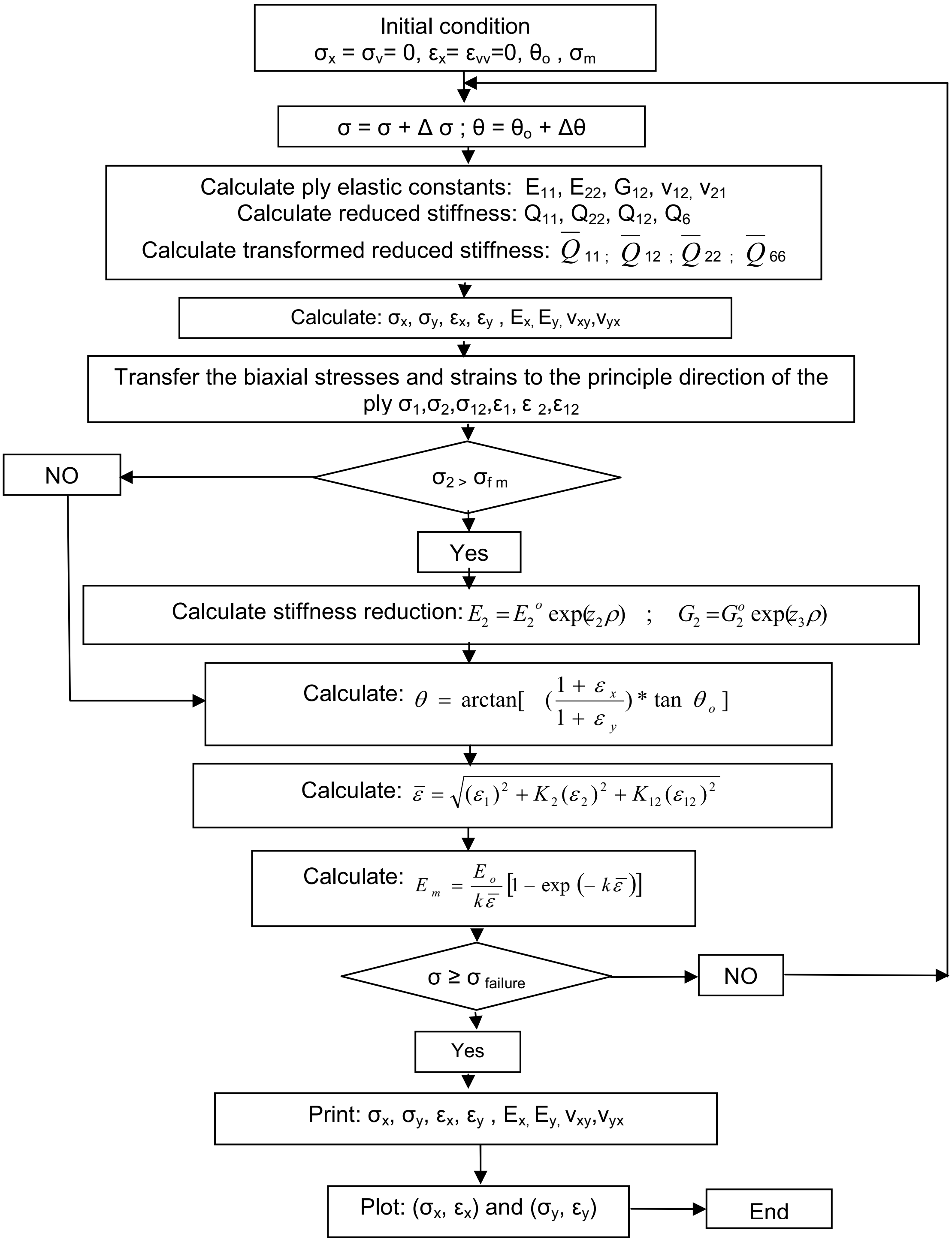

Figure 4: Flow chart of the model iteration.

These calculations were repeated in each step by small increments of applied stress ($\Delta\sigma$) and angle ply ($\Delta\theta$). After that, the condition of matrix cracking ($\sigma_2 > \sigma_{f\,m}$) was checked. If this condition was satisfied, the stiffness reduction must be calculated using equation (14-16) and continued to calculate the fiber rotation, equivalent strain and secant matrix modulus using equations (17-22). The iteration procedure will stop when the increment load reached the failure load. Finally, the output data such as stresses, strains, modulus of elasticity and poisons ratios were printed. The plots stress – strain curves can be drawan from these data and compared with the experimental curves. This procedure was implemented in a Visual Basic computer program. The model was tested using experimental data of stress- strain tests from the literature [2, 16, 19, 21]. The material properties of the fibres and resins and non-linearity fitting parameters were deduced in the model according to the type of fibre and resin as shown in Table (1).

Table 1: Material properties and parameters used for modelling procedure [1, 2,16].

Resin or Fibre	E_m or E_f MN/m^2	σ_m MN/m^2	ν_m or ν_f	Z_1	Z_2	Z_3	K1	K
Epoxy	3000-4000	40-50	0.34	0.67	0.67	0.67	1	100-300
Phenol	850	10-20	0.33	0.67	0.67	0.67	1	100-300
Polyester	3000-4000	40-50	0.38	0.67	0.67	0.67	1	100-300
E-glass	70000-76000	---	0.253	---	---	---	-	----
Kevlar	70000-135000	---	0.21	---	---	---	-	----
Boron	400000	----	0.2	---	---	---	-	----

RESULTS AND DISCUSSION

Results of the modelled stress-strain curves were compared with experimental curves chosen from different references in the literature. Figures (5-7) show plots of experimental and predicted stress-strain curves for ± 45° angle ply glass/polyester, Kevlar /polyester and glass / epoxy laminates respectively.

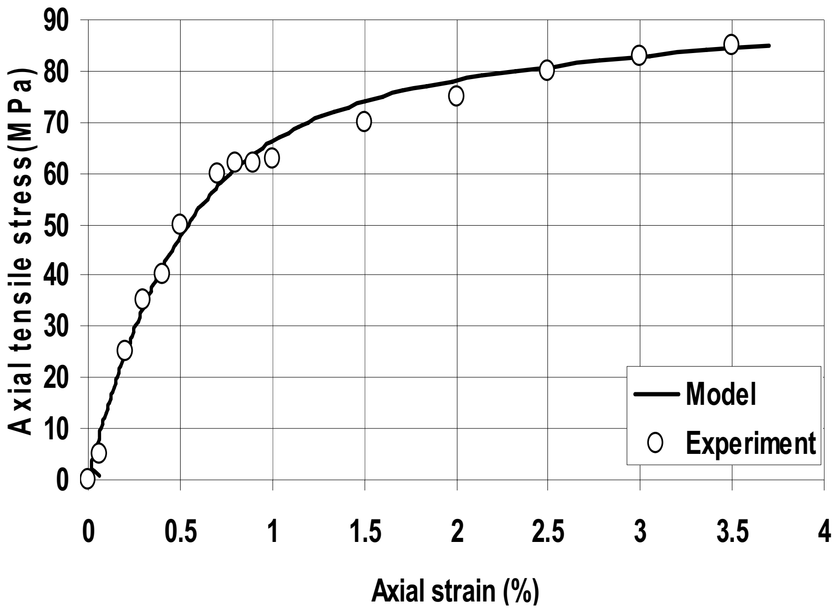

Figure 5: Modeled and experimental stress-strain curves for ± 45° glass/polyester angle ply laminate under tensile loading [16].

The experimental data of the Figures (5) and (6) were taken from references [16] and for Figure (7) were taken from [19]. It can be noted that the curves of the three laminates are similar and exhibited the same behaviour. Initially they are linear and then become nonlinear up to failure. A good agreement was observed between the modelled and experimental results. This implies that the proposed nonlinearity parameters used in the model are adequate.

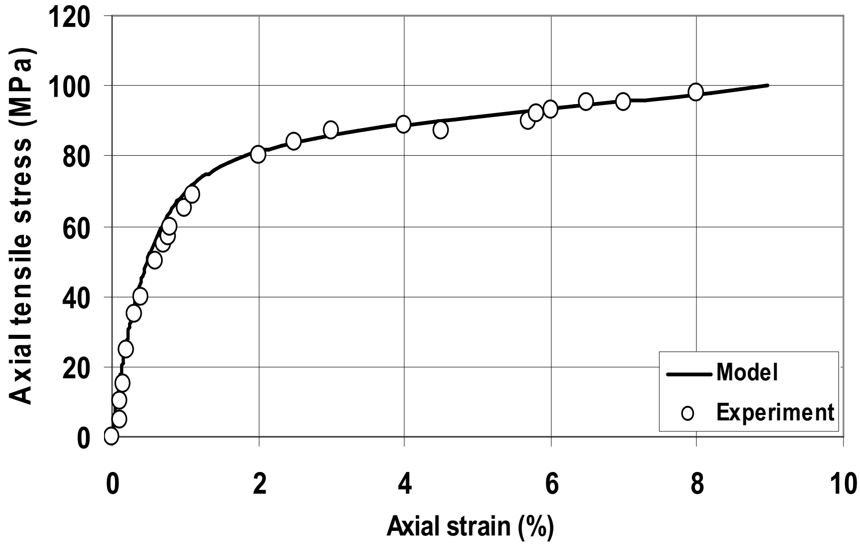

Figure 6: Modeled and experimental stress-strain curves for ± 45° Kevlar/polyester angle ply laminate under uniaxial tensile loading [16].

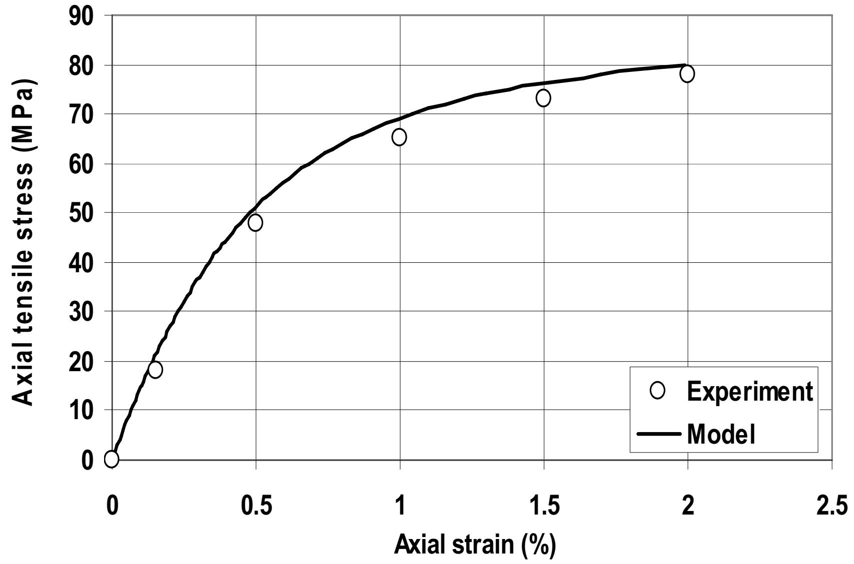

Figure 7: Modeled and experimental stress-strain curves for ± 45 angle ply laminate made from glass/ MY750HT917/DY063 under uniaxial tensile loading [19].

Figures (8-10) show comparison between three stress-strain curves for present model, Pitito's model [2] and excremental results [21]. From these comparisons, it can be seen that all the curves exhibited nonlinearity behaviour and there is a reasonable agreement between the experimental and the modelled curves. However, a discrepancy between the present model and Petit model at high strain to failure can be observed. This is probably related to the differences between the formulation routines of the models. Petit model is based on the linear lamination theory and incrementally applied the average laminate stresses ($\Delta\sigma_x^o\ \Delta\sigma_y^o\ \Delta\ \sigma_{yx}^o$) as non linear material.

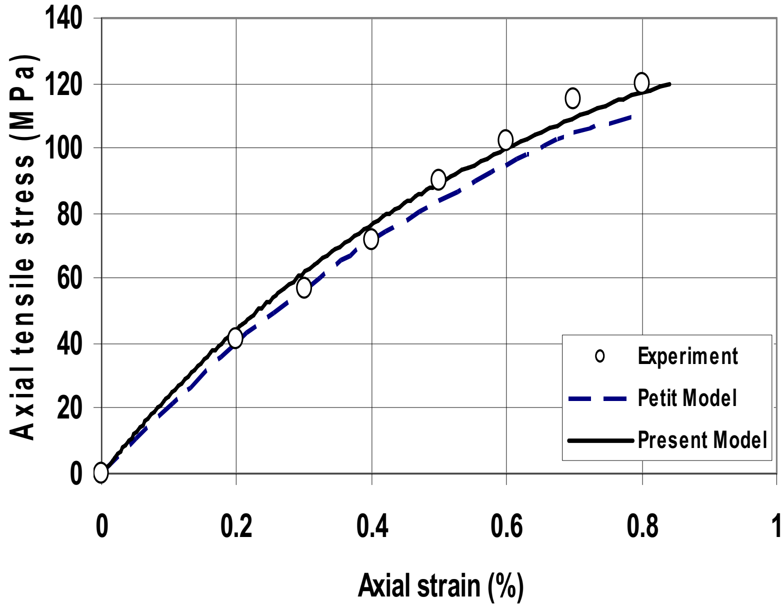

Figure 8: Modeled and experimental tress-strain curves for ± 60° boron/epoxy angle ply laminat under uniaxial tensile loading [2, 21].

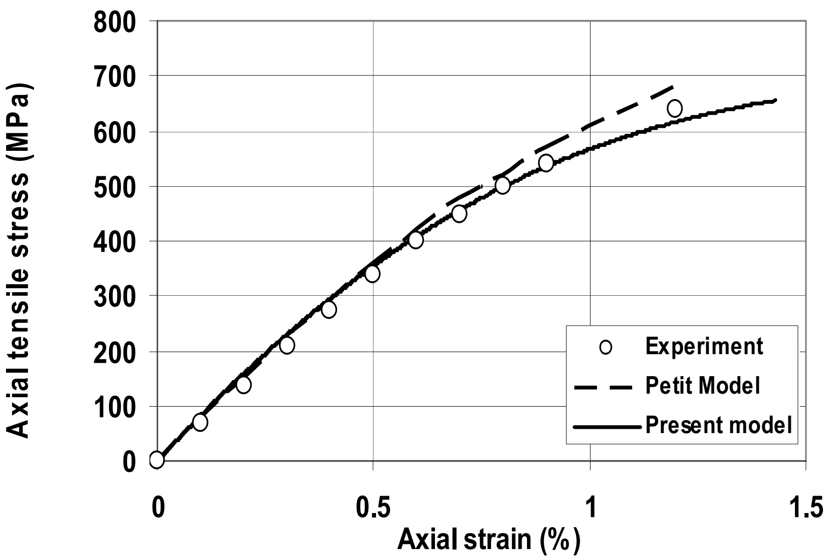

Figure 9: Modeled and experimental stress-strain curves for ± 30° boron/epoxy angle ply laminate under uniaxial tensile loading [2, 21].

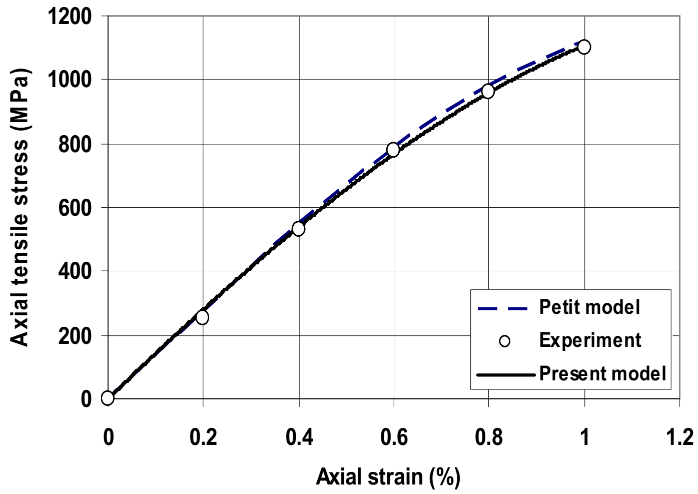

Figure 10: Modeled and experimental stress-strain curves for ± 20^o boron/epoxy angle ply laminate under uniaxial tensile loading [2, 21].

Furthermore, the modelled results were compared with experimental results of biaxial loading tests from [16] as shown in Figures (11 and 12). Figure (11) shows stress-strain curves for a ± 55º filament wound glass/polyester composite pipes subjected to biaxial loading (internal pressure) at room temperature whereas in Figure (12), the pipe was subjected to pure axial loading in wet and hot condition at 160 ºC. It was observed that the curves in both cases exhibited non linear behaviour which is due to the effect of matrix cracking, fibre rotation and yielding of the resin or combination of any of them. Experimental evidence by Rosenow [21] indicated that the failure of filament composite pipe under various internal pressures was characterised by matrix micro cracking followed by penetration of the fluid out of pipe wall resulting in a considerable plasticizing effect on the resin.

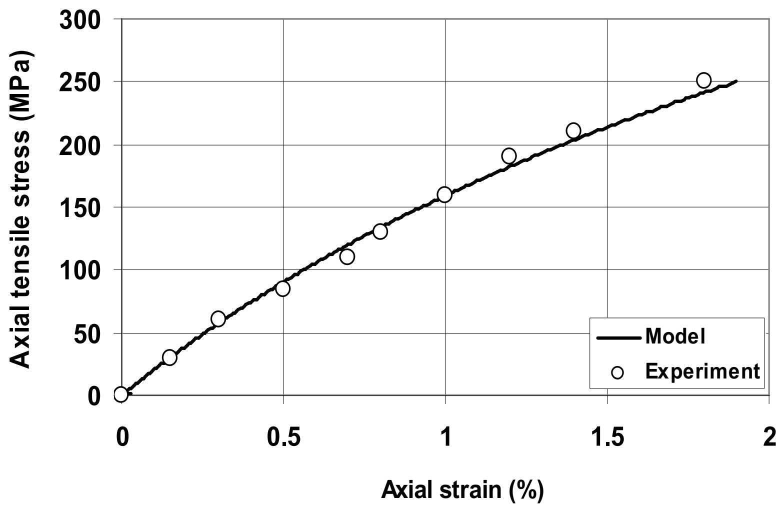

Figure 11: Modeled and experimental stress-strain curve for ± 55^o filament wound glass/polyester composite pipe subjected to internal pressure at room temperature [16].

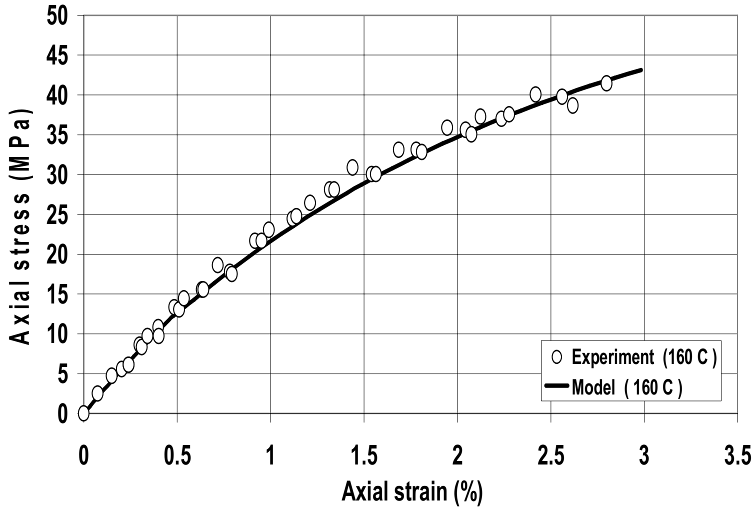

Figure 12: Modeled and experimental stress-strain curves for ± 55^o filament wound glass/ phenolic (PSX) composite pipe under pure axial loading in wet condition at 160C^o [16].

CONCLUSIONS

The results showed that the model developed is capable of predicting the nonlinear behaviour of composite materials. The ability to observe the nonlinearities in stress-strain behaviour is advantageous in interpreting material properties. It was found that the nonlinearity was strongly enhanced by matrix cracking, fibre rotation and yielding of the resin. The model is found to be in good agreement with experimental results as well as other analytical results

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NOMENCLATURE

c, s	Cosθ and sinθ
E_{11}, E_{22}, G_{12}	Longitudinal, transverse and shear moduli in principal directions.
E_f, E_m	Fibre and matrix Young's moduli.
E_2 , G_2	Effective transverse and shear moduli in principal directions.
$E_2^{\,o} , G_2^{\,o}$	Initial transverse and shear moduli in principal directions.
E_o	Initial tangent modulus.
$E^o{}_m$	Initial matrix Young's modulus.

G_f, G_m	Fibre and matrix shear moduli.
k_{22} , k_{12}	Constant parameters
k	Empirical parameter
k_2, k_{12}	Concentration factors for the transverse and shear loadings
[R]	Transformation equation.
$Q_{11}, Q_{22}, Q_{12}, Q_{66}$	Reduced stiffness of the undamaged ply.
$\tilde{Q}_{11}, \tilde{Q}_{22}, \tilde{Q}_{12}, \tilde{Q}_{66}$	Effective reduced stiffness.
$\overline{Q}_{11}, \overline{Q}_{22}, \overline{Q}_{12}, \overline{Q}_{66}$	Transformed reduced stiffness.
V_f, V_m	Fibre and matrix volume fractions.
ν_{12}, ν_{21}	Poisson's ratios in principal directions.
ν_f, ν_m	Poisson's ratio for glass fibre and matrix.
ν_{xy} ν_{yx}	Poisson's ratio in x and y directions
$\sigma_x,^* \sigma_y^*$	Stresses in coordinate systems
$\sigma_1,^* \sigma_2^* \tau_{12}^*$	Longitudinal, transverse and shear stresses in principal directions.
σ_{fm}	Failure strength of the matrix.
$\varepsilon_x, \varepsilon_y$	Strains in coordinate systems
$\varepsilon_1, \varepsilon_2, \varepsilon_{12}$	Strain in principle directions
$\bar{\varepsilon}$	Equivalent strain.
ρ	Crack density.
ξ_{12}, ξ_{22}	Adjustable parameters
η_{12}, η_{22}	Halpin Tsai parameters
z_2, z_3	Dimensionless constants.
θ_o	Initial filament winding angle to the tube axis.
θ	Filament winding angle to the tube axis.