STRESS ANALYSIS OF SANDWICH STRUCTURE USING FINITE ELEMENT METHOD

Tarik S. Al-Shhbony and Said H. S. Eshtewi

Applied Mechanics Division Department of Mechanical Engineering
Al-Fath University Tripoli Libya
B. O. Box 13648, Tripoli Libya
Email; seshtewi@ hotmail.com

الملخص

تعد ألواح الطبقات البينية حالة خاصة من الصفائح المركبة وهي عبــارة عــن طبقتــين رقيقتين قويتين ذات كثافة عالية ويفصل بينهما طبقة خفيفة الوزن تسمى باللب. وجهي الطبقات البينية يصنعان من مادة لها خواص متجانسة وغير متجانسة واللب قد يكون على شــكل خليــة النحل. تم تحليل الإزاحة والاجهادات للطبقات البينية المعرضة للحمل في المستوى. كمــا تــم إشتقاق المعادلات التفاضلية لاتزان الأحمال المطبقة وحلها باستخدام برنامج العناصر المتناهيــة لحساب الاجهادات.

استخدمت صفيحة مستطيلة الشكل لمقارنة الحل باستخدام طريقة جليركن (Galerkin). تم إستخدام طريقة جليركن مع برنامج جاهز كخطوة أولية للتأكد من دقة البرنامج الحالى وكانت النتائج متفقة. بهذا استخدمت طريقة جليركن لتحليل الطبقات البينية لعدة حالات من الشروط الحدودية للتحميل في مستوى لحساب الإزاحة والاجهادات في الطبقات البينية. يمكن استخدام أنواع مختلفة من المواد للأوجه واللب في هذا البرنامج و يمكن استخدامه في تحليل الصفيحة الواحدة والصفائح المركبة. النتائج المتحصل عليها للإزاحة والاجهادات عند التحميل في مستوى مع اختلاف مادة الأوجه واللب كانت جيدة جدا مقارنة مع برنامج العناصر المتناهية الجاهز.

ABSTRACT

Structural sandwich panels can be considered as a special type of composite laminate where two thin, stiff, strong and relatively dense faces are separated by a thick, light weight and compliant core material. The sandwich faces are made of orthotropic or isotropic material and the core is of a honeycomb type. An analysis of displacement and stress in a sandwich plate structure subjected to in-plane loads was carried out. The differential equations of equilibrium for in-plane load elasticity and a final numerical solution were prepared and a finite element program was written for estimating the stresses in the structure.

A rectangular plate was used to compare the results obtained using Galerkin's method with those obtained by using the commercial finite element software to check that the program works well as a first step and good agreement was achieved. Accordingly, the Galerkin's method is used to solve sandwich structure for different boundary conditions and in-plane loads to evaluate the displacement and stresses in the sandwich structure. The program can be used for different material of faces and core and also for composite laminated structure. Displacement and stress results for in-plane loading for different kind of materials of core and structure show very good comparison with other finite element program.

KEYWORDS: Sandwich; Faces and core; Honeycomb; Plate; Orthotropic; Laminate.

INTRODUCTION

The sandwich construction is growing up to replace the other configuration structures around the world and will continue to be in demand. It is playing an increasingly very important role in structures because it has high flexural stiffness-to-weight ratio compared with the others. By means for a given set of mechanical and environmental loads, sandwich construction often results in a lower structural weight than do other configurations.

The first research paper concerning sandwich construction was due to Marguerre [1] in 1944. The early theoretical work on the behavior of rectangular sandwich panels subjected to lateral loads was applied only to uniform loads and simply supported edge conditions. By the mid 1960s, efforts in sandwich research had spread widely [1]. A fairly complete bibliography describing all publications regarding sandwich construction before 1966 is provided in [2]. Ha [3] and Bert [4] gave an overview of finite element and analytical methods applied to sandwich. Karbhari [5] provides an overview of the use of composite-sandwich usage for the twenty-first century.

The applications of sandwich have been used in all industry that need high-strength-to-weight ratio or to transmit heat or to be an insulative barrier or to absorbing mechanical and sound energy. Bitzer [6] gave an excellent overview of the uses of honeycomb-core materials and applications. He concludes that honeycomb-core sandwich construction is widely used on both commercial and military aircraft. More recently, different sandwich constructions are being used increasingly in civil engineering such as bridge decks. Woldesenbet [7] investigated the use of sandwich construction for low cost and emergency housing. Kujala and Tuhkuri [8] investigated the use of steel-corrugated panels for superstructures in ships and they found that sandwich structures were 40-50% lighter than the conventional steel construction.

Symmetric sandwich structures with constant core and face thicknesses can give simple design configuration that will provide sufficiently lightweight panels for many purposes. However if we need sandwich structure that are highly optimized with respect to weight, it is often necessary to use asymmetric sandwich panels with variable core and face thicknesses [9-10]. Very little research has been done to investigate such geometrical change effects in these sandwich panels.

The objective of the work presented was to develop a finite element program and to provide an in depth understanding of the isotropic and orthotropic sandwich structure. The model of finite element is based on laminated plate. The finite element technique has been used primarily to calculate the stresses in the plate model to check the program works in good results. The data of this finite element model were compared with other data for the sandwich panel with different properties. In addition the displacements and stresses distribution and other fundamental information have also been included in this paper.

FORMULATION OF FINITE MODEL

Finite element methodology is used in this work to analyze laminated sandwich plate by Galerkin's method [11-12]. The group discretized into composite rectangular elements, where each element consists of three rectangular laminate. The procedure carries the analysis from the original governing equation to the final numerical solution. The differential equations of equilibrium in plane elasticity [13-14] are given as:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + f_x = 0 \tag{1}$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + f_y = 0 \tag{2}$$

The stiffness matrix for three rectangular elements can be generated as follows:

$$[k^e]_c = \sum_{k=1}^{n} \sum_{I=1}^{n} W_{nk} W_{nl} [[B]^T [A_{ij}][B] \det[J]](\xi_{nk}, \eta_{nl}) \tag{3}$$

The load matrix for three rectangular element can be generated as follows:

$$\{F^e\}_c = \sum_{k=1}^{n} \sum_{l=1}^{n} W_{nk} W_{nl} [[N]^T [f] t \ \det[J](\xi_{nk}, \eta_{nl}) \tag{4}$$

The total global structural matrix is obtained by using the direction stiffness method concept [11,14-15]

$$[K] = \sum_{e=1}^{n} [K^e]_c \tag{5}$$

$$\{F\} = \sum_{e=1}^{n} \{F^e\}_c \tag{6}$$

Where n is the total number of degrees of freedom of the structure which is equal to the number of degrees of freedom per node times the total number of nodes.

$$\{U^e\}_c = [K]^{-1} \{F\} \tag{7}$$

$$\{F\} = [K]\{U^e\}_c \tag{8}$$

Applying the boundary conditions to determine the unknown global structure nodal displacements and then the stresses in the global x and y directions for each lamina in the composite elements [1, 12, 15].

$$[\{\sigma\}_e]_K = [\overline{Q}_{ij}]_K [B]\{U^e\} \tag{9}$$

$\{\sigma\}_e$: The stress matrix in the element, $[\{\sigma\}_e]_K$: The stresses in element in each lamina,

$[B]$: The strain matrix, $\{U^e\}$: The displacement matrix of the composite element,

$[\overline{Q}_{ij}]$: The elasticity stiffness matrix in each lamina.

RESULTS AND DISCUSSION

Comparative finite element analyses have been conducted for laminate sandwich structure with constant face/core thickness. For the purposes of the present paper, selected comparative results obtained for symmetric sandwich panel configuration are presented. The properties of material and dimensions for the faces and core are given in Table (1). The boundary conditions are fixed in one end and in-plane load of P=2.5 kN/m in the other end. The other tow ends are free. The finite element analyses were performed using four-nodes for 25 elements.

Table 1: Mechanical properties and dimensions of honeycomb core and faces

Faces			Core		
Material	Parameter	Value	Material	Parameter	Value
Boron / Epoxy	E_1	$2.1 x 10^5$ MPa	Aluminum Alloys 2024	E	$7.23 x 10^4$ MPa
	E_2	$0.2 x 10^5$ MPa		V	0.33
	V_{12}	0.3			
	G_{12}	$0.69 x 10^4$ MPa		G	$2.71 x 10^4$ MPa
	t	0 0.2 mm		T	5 mm
Dimension of Laminate plat			a = 250 mm		b = 250 mm

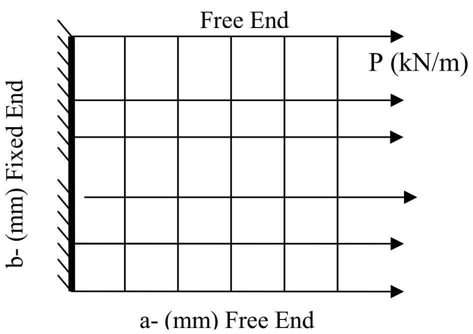

Figure 1: Sandwich boundary conditions and load

Figure (1) shows the case study of honeycomb sandwich structure fixed from one end, where the other side carrying forces per unit length. The two other ends are free.

A program of 1800 statements with Fortran Power-Station was used and run. Mesh generation and summation the properties of sandwich panel were used to computed the element stiffness matrix and load matrix and then the global matrix. By applying the load and boundary condition for this case study the system of simultaneous of linear algebraic equation were performed. Solution of these equations will evaluate the nodal displacement and the stresses in each layers the honeycomb structure (faces and core) and Figure (2) shows the flow chart program in this work.

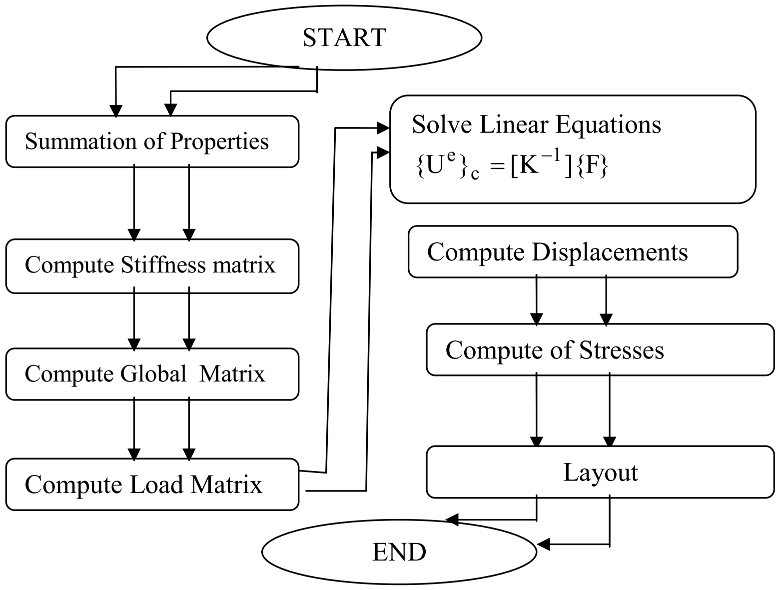

Figure 2: Flow chart diagram of the program

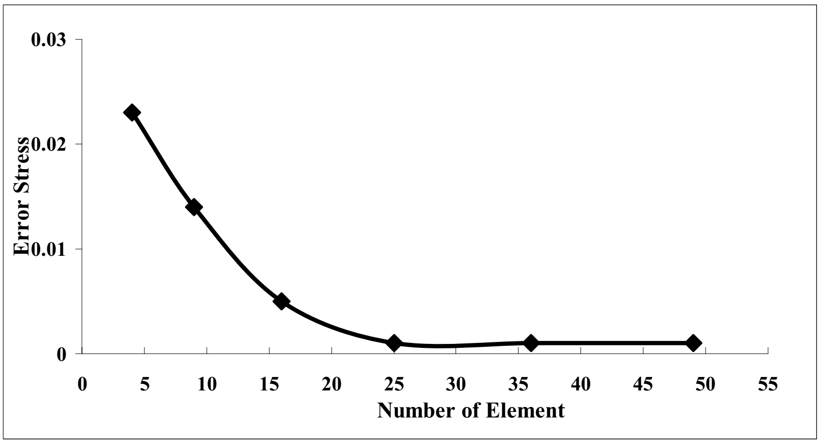

Figure 3: Error stress versus noumber of elements

Figure (3) shows the convergence of error stress as the number of elements is increased up to twenty-five elements then no change in the error difference of the stresses for the forty-eight elements used. Hence, where the difference gets stable then this number of elements (which is 25) will be used throughout the data for the finite element program to save time and effort. By using more elements than twenty-five is no meaning to west of time.

Figure (4a) shows the sandwich panel as predicted by the finite element analyses in this work and reference data [16] for the contours of u_x displacement. They agreed that the peak occurs at the end of applied in-plane load. Figure (4b) shows the same data on x-y coordinates and the two curves are the same. Also for the v_y displacement is nearly the same and not shown here.

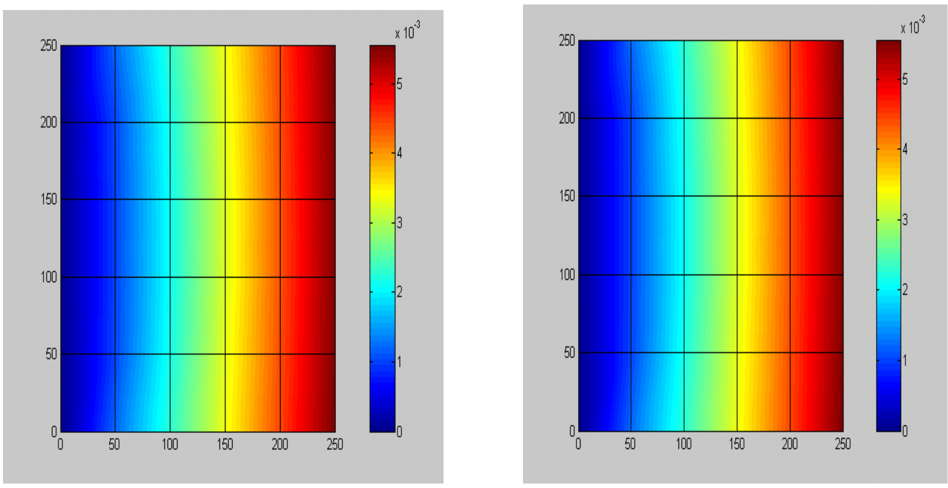

Figure 4a: u_x Displacements (Left side present analysis and right side Ref. 9)

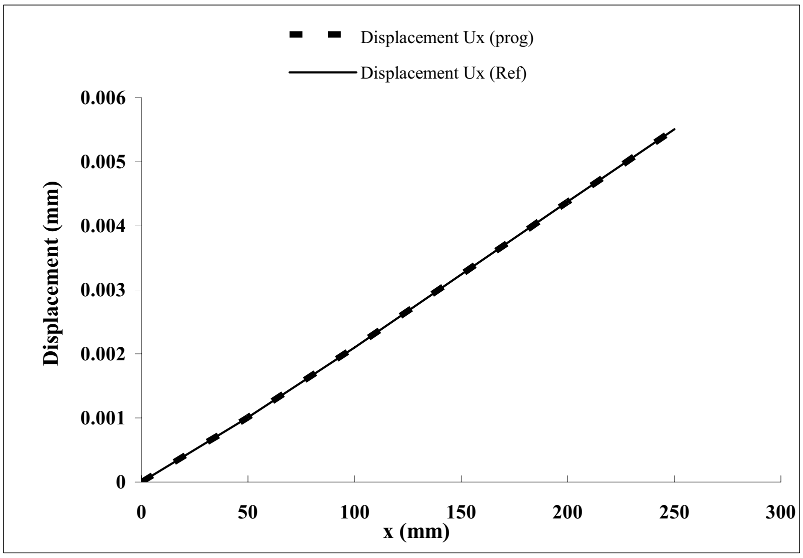

Figure 4b: u_x displacements (present analysis compared to Ref. 9)

The normal stresses contours with different colures for faces in x-direction obtained from reference data and this program generally compared very well and was shown in Figure (5a). The same results in Figure (5b) were done to them on x-y plane and the two curves for stress are nearly are the same.

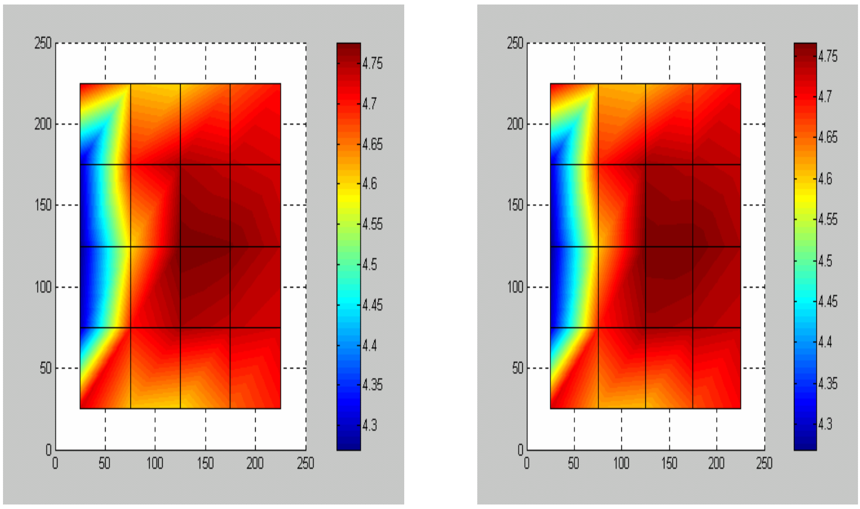

Figure 5a: Stress σ_{xf} in face (Left side present analysis and right side Ref. 16)

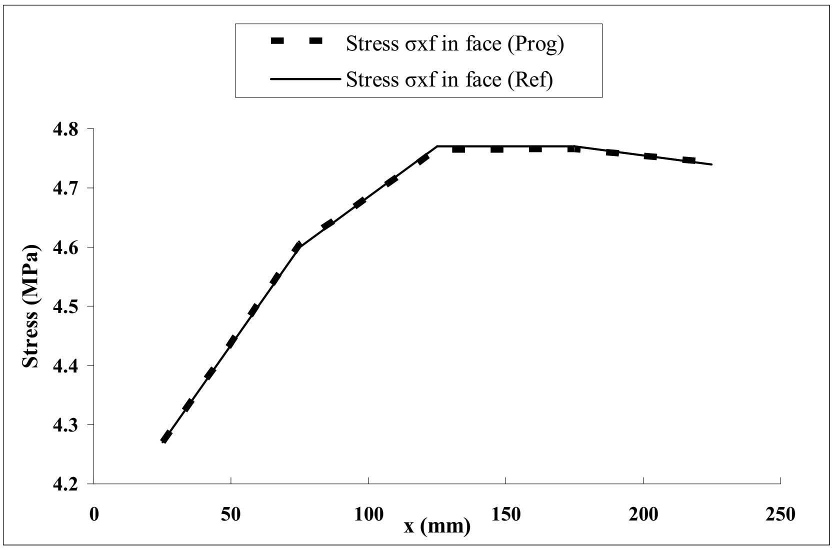

Figure 5b: Stress σ_{xf} in face (present analysis compared to Ref. 16)

The normal stresses contours with different intensity colure in core in x-direction obtained from reference data and this program generally compared very well were shown in Figure (6a) the other Figure (6b) indicates same data on x-y coordinate.

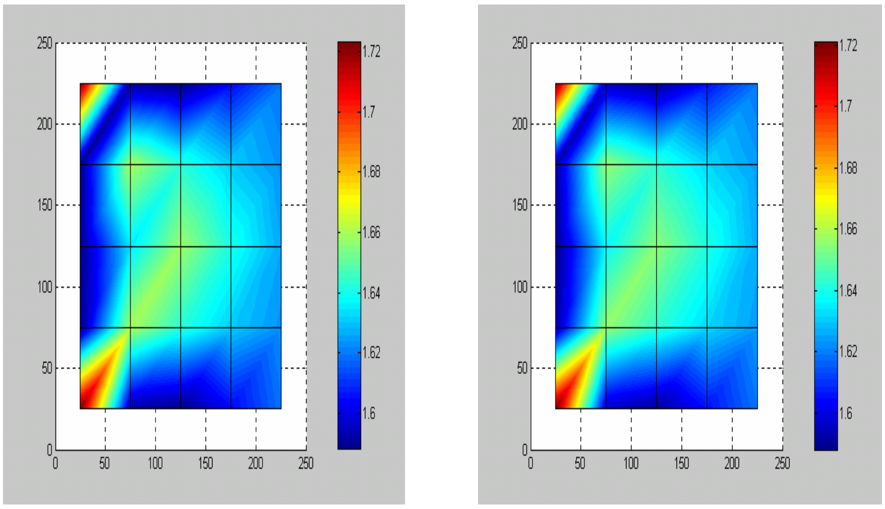

Figure 6a: Stress σ_{xc} in core (Left side present analysis and right side Ref. 16)

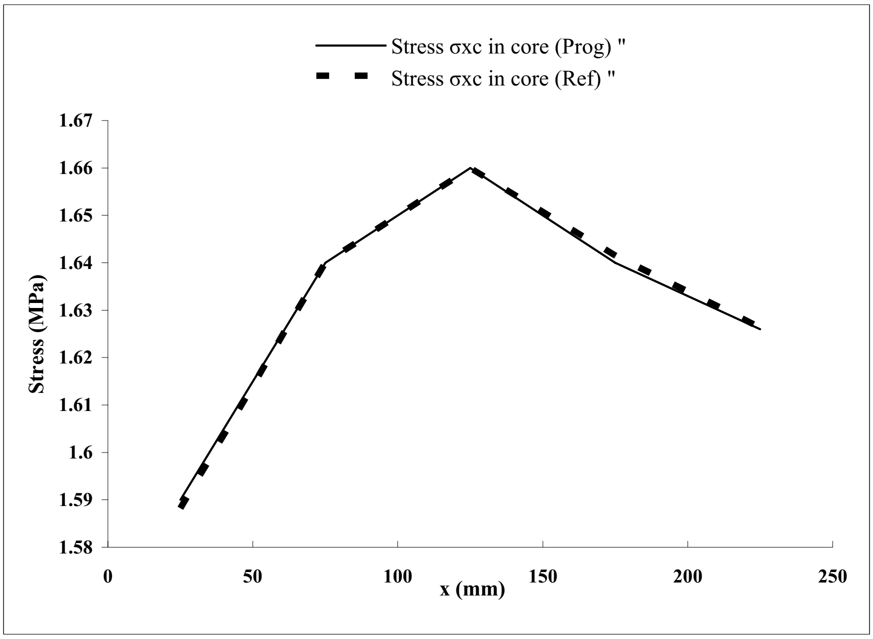

Figure 6b: Stress σ_{xc} in core (present analysis compared to Ref. 16)

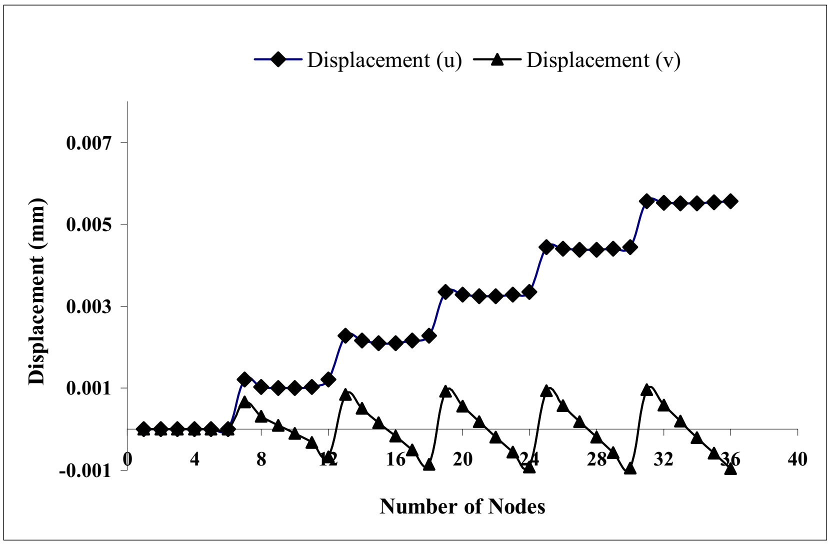

Figure 7: Displacement distribution (u_x,v_y)

Figure (7) shows the distribution of the two displacements with number of elements and as expected these are zero the fixed end and peak points. Also Figures (8a and 8b) show the distribution of stresses (σ_x and σ_y) and number of elements in the face and core where are high near the fixed end for the faces as expected. Shear stresses distribution with number of elements is shown in Figure (9) for face and core and the peak value is in the core, which is typical for sandwich structures.

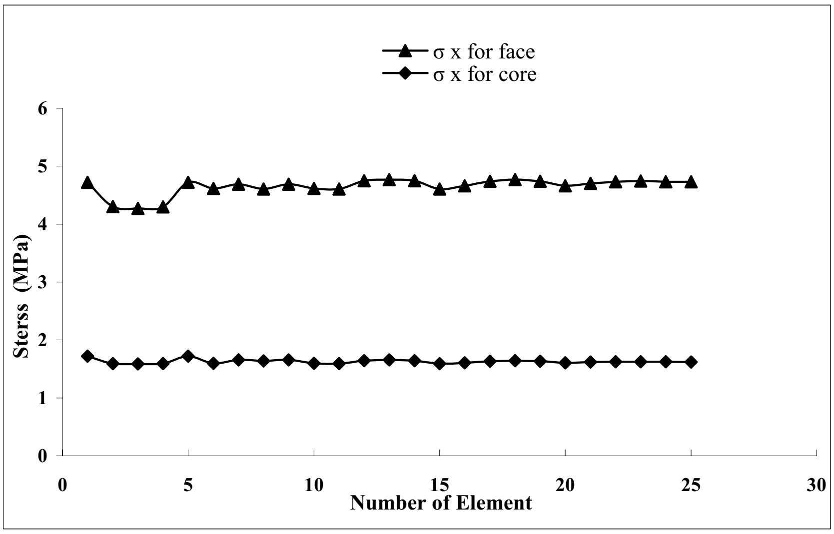

Figure 8a: Distribution of normal stress (σ_x) in core and face

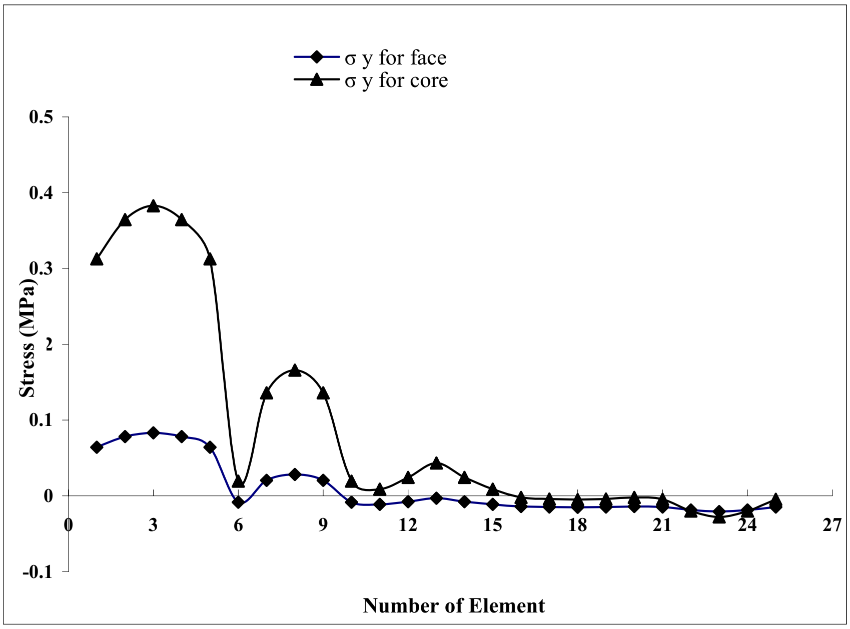

Figure 8b: Distribution of normal stress (σ_y) in core and face

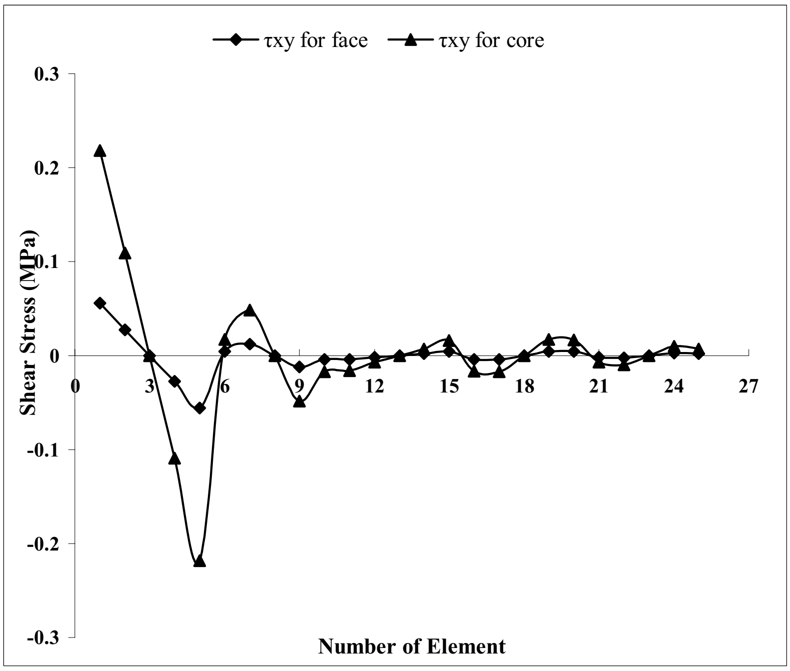

Figure 9: Distribution of shear stress in face and core

Figure (10a) shows the difference of the shear stresses calculations of this work and data reference in graph of errors for the same number of elements. The maximum difference is about 1.1×10^{-5} The results of displacement in Figure (10b) show that the error will be less than 0.23% over data reference which shows a very good agreement, therefore, can be used for other cases.

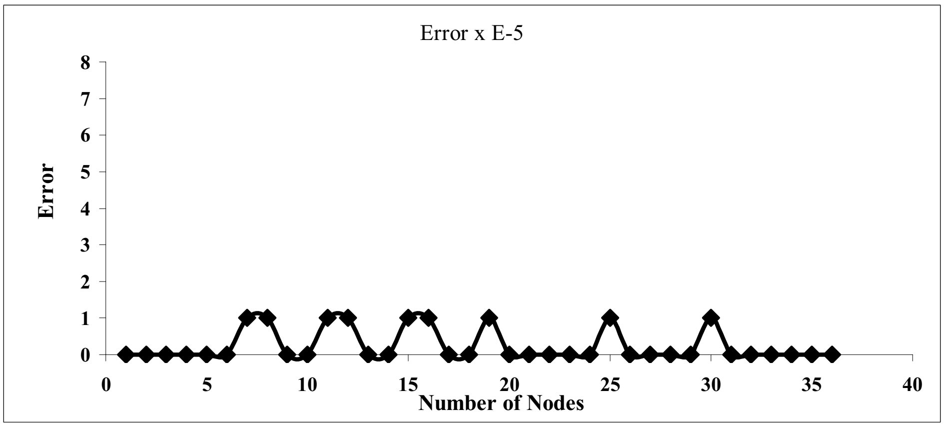

Figure 10a: Error between present analysis and Ref. 16

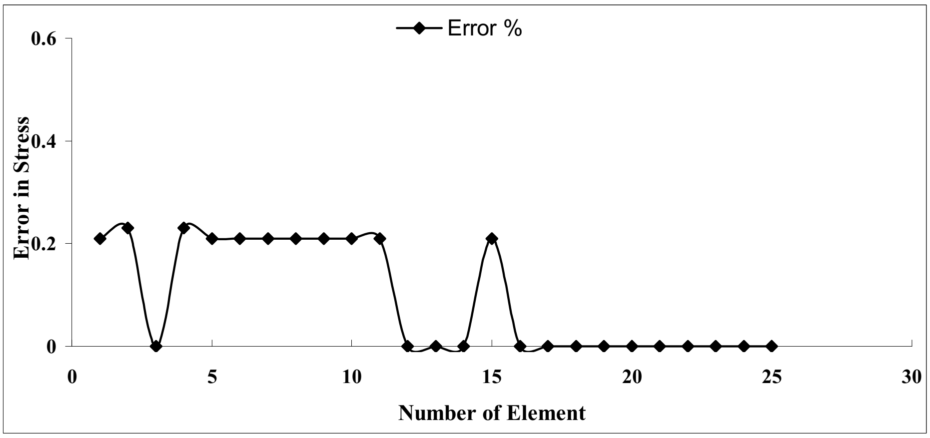

Figure 10b: Error between present analysis and Ref. 16

CONCLUSION

Sandwich structures are well suited for transfer of overall bending and shearing loads. Localized shearing and bending effects induced severe through-thickness shear and normal stresses. The stress components can be of significant magnitude and may in many cases approach or exceed the allowable stresses in the core and faces.

The results obtained for the laminate plate were compared with results obtained from other finite element data. It was found that increasing the number of elements beyond twenty five does not cause any change in the stresses values. The comparison for the stresses and displacements results demonstrated a close match between the results obtained using the two finite element methods. The calculations of displacements and stresses yield results in good agreement with the available referenced data; an indication that the finite element program works very reasonably and that it can be developed and used for other boundary conditions, applied loads and different material properties.

ACKNOWLEDGMENT

The authors wish to acknowledge with deep appreciation the assistance provided by MSc. Eng. B. Al-shhbony and Eng. T. Keary during prepare of this work.

REFERENCES

[1] J. R. Vinson, “The Behavior of Sandwich Structures of Isotropic and Composite Materials”, Technomic, Lancaster,PA,1999.

[2] J. R Vinson and S. Shore, “Bibliography on Methods of Structural Optimization for Flat Sandwich Panels,” Naval Air Engineering Center Report NAEC-ASC-1082, April 15, 1965.

[3] K. H. Ha, “Finite Element and Sandwich Construction: A Critical Review, “in Sandwich Construction I, Proceeding of the First Conference on Sandwich Construction, United Kingdom, 1989.

[4] C. W. Bert, “Part I, and Part II”, Shock and Vibration Digest Vol. 23, 1991,pp. 3-14:and pp.19-21.

[5] V. M. Karbhari ”Application of composite Materials to the Renewal of Twenty-First Century Infrastructure,” Proceeding of the Eleventh International Conference on Composite Materials, Gold Coast, Australia, 1997.

[6] T. N. Bitzer, “Honeycomb Materials and Applications” in Sandwich Construction 2, Proceeding of the Second International Conference on Sandwich Construction, United Kingdom, 1992.

[7] E. Woldesenbet and J. R. Vinson, “Sandwich Composite Structure for Low-Cast and Emergency Housing, “in Sandwich Construction 3, Proceeding of the Third International Conference on Sandwich Construction, United Kingdom, 1996.

[8] K. Kujala and J. Tuhkuri, “All-Steel Corrugated-Core Sandwich Panels for Ship Structures,” in Sandwich Construction 3, Proceeding of the Third International Conference on Sandwich Construction, United Kingdom, 1996.

[9] O. T. Thomsen, and J. R., “Modeling of Tapered Sandwich Panels Using a High-Order Sandwich Theory Formulation” AIAA JOURNAL, Vol. 40, No. 9, September 2002.

[10] O. T. Thomsen, “Theoretical and Experimental Investigation of Local Bending Effects in Sandwich Plates,” Composite Structures, Vol. 1, 1995.

[11] S. Singiresu, “The Finite Element Method in Engineering”, 3rd Edition University Miami Florida, USA 1999.

[12] A. Barna, A. M. Szabo Asce and G. C. Lee, Stiffness Matrix for Plates by Galerkin’s Method Journal of the Engineering Mechanical Division, Vol. 95, June 1969.

[13] S. P. Timoshenko and J. N. Goodier, “Theory of Elasticity”, Third Edition, International Student edition, 1982, McGraw-HillInt.

[14] Naffati M. Werfalli "Lectures in Finite element analysis", Course No. ME605, (2002), Mechanical Engineering Department, Applied Division, Al-Fath University.
[15] S. Moaveni "Finite Element Analysis"m Minnesota State University, 1999.
[16] Reference Comparable Data Available, "MSC. Visual Nastran 2001, Version 70.7

Nomenclature:

A_{ij}	Extensional stiffness	a	Panel Length
b	Panel With	B	Strain Matrix
E	Young Modules	F	Load Matrix
G	Modules of Rigidity	J	Jacobion Matrix
K	Stiffness Matrix	N	Shape Function
t	Thickness of Sandwich	U	Displacement Matrix of Sandwich Element
u_x	Displacement in x-Direction	ζ, η	Guess Points
θ_{ij}	Elasticity Stiffness Matrix in Each Lamina		

σ_{xf} Normal Stress in Face σ_{xc} , Normal Stress in Core , τ_{xy} Shear Stress