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Contents

PARAMETRIC STUDY OF CORRUGATED PIPES AND BELLOWS USING A MIXED FINITE ELEMENT METHOD.

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1. INTRODUCTION.

Study of the flexibility of corrugated pipes started as early as 1932, when Donnell [1] used virtual work in obtaining a reduced modulus of elasticity for corrugated pipes in comparison to a smooth pipe having the same thickness and mean radius. In general, the study that followed is either a theoretical or numerical development dealing with toroidal shells [2-10], or an experimental study dealing with specific types of bellows, which sometimes were accompanied by an approximate theoretical approach for verifying the results [11-14]

In references [15,16], the writers developed a mixed finite element method for the linear analysis of shells of revolution under the action of symmetric and non-symmetric loading. Good results were obtained for the wide range of problems which tested the accuracy of the solution in comparison to the analytical or exact values. In this paper the mixed finite element method is used in solving the problem of corrugated pipes and pipe expansion joints (Bellows), where relationships for the axial stiffness of pressurized and non-pressurized bellows, as well as their bending stiffness are obtained. Accordingly a corrugated pipe can be solved as a beam or an arch if it is curved.

2. PARAMETERS OF THE PROBLEM.

The meridional curve of the corrugated pipe is considered to be sinusoidal in shape as shown in figure 1, where the parameters L, R, h and c are the controlling geometrical parameters of the problem.

The mean radius (R) is fixed to 100.0 and the ratios h/R, c/R and L/R are given as

variables, modulus of elasticity (E) is taken as unity, and Poisson's ratio as 0.25, which is thought to be reasonable for most metals. The following are the considered variations in the parameters of the problem:

h/R = 0.01, 0.015, 0.02

c/R = 0.0, 0.1, 0.15, 0.2

L/R = 1.0, 2.0, 3.0, 4.0

where

h = shell thickness

c = half depth of corrugation

R = mean radius

L = wave length of corrugation

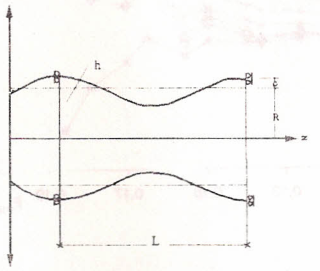

Fig. 1 Corrugated pipe of sinusoidal meridian showing geometrical parameters

The parametric study of expansion joints is dealth with using a limited number of test cases. This is due to the limit of computer time as well as reducing the time taken in analysing the results.

3. AXIAL STIFFNESS

For the axial stiffness, the given parametric cases were solved when the bellows was under a unit axial displacement with no pressure, and when a unit internal pressure was present.

a) Axial Stiffness of Bellows with Internal Pressure

Where the corrugated pipe is under internal pressure, hoop forces or hoop stresses become very important, they are higher in value when compared to the meridional forces or stresses. It should also be noted that the hoop forces differ in value at the crest and at the throat of the shell, in addition the maximum hoop forces tend to occur about mid-way between the crest and the throat. Bending stresses are noticed to be very small in comparison to the membrane stresses.

Figure 2 shows the changes in the values of hoop forces at the crest and the throat in relation to the changes in c/R , h/R and L/R

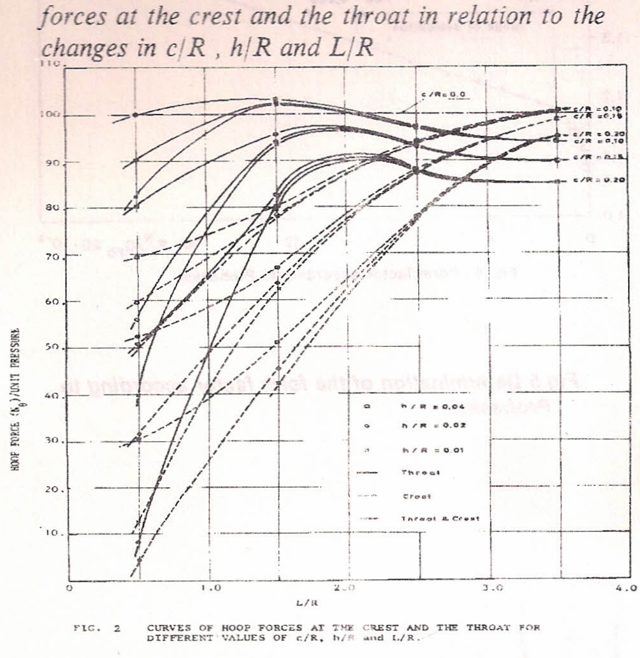

FIG. 2 CURVES OF HOOP FORCES AT THE CREST AND THE THROAT FOR DIFFERENT VALUES OF c/R, h/R and L/R.

They are equal for the basic shape which is a cylinder and then diverge with the changes in the governing parameters. The curves given in figure 2 show that for à ratio of L/R between 2 and 4., the changes in h/R make slight or negligible change in the hoop forces. Also at the ratio ($L/R \simeq 3.2$) hoop forces at both the crest and throat start to become equal or very close to each other.

This point is termed as optimal as far as the dominating hoop forces are concerned, and the range of L/R = 3 to 4 becomes the preferred range

The second step is to study the behaviour of the maximum hoop forces which as stated earlier lie between the crest and the throat. These forces are also found to be very high for small ratios of L/R in comparison to the forces at the crest and throat, and tend to become very close to them at the range (L/R = 3 to 4).

Figure 3 shows the behaviour of the maximum surface hoop stresses for the range (L/R = 3 to 4 and c = .0 to .2) against the change in the ratio h/R. The curve presented in figure 3 is a third order polynomial, giving the maximum hoop stresses for the ranges of given parameters with a maximum error of ∓ 4.4%, and the equation of the curve is given as:

$$\sigma_{\theta max} = p\,(203. - 1300.\ h/R + 329000.h^2/R^2 - 2860000.\ h^3/R^3) \qquad (1)$$

where p is the intensity of the internal pressure.

The third step is to find a relationship governing the axial stiffness of these corrugated pipes under pressure. The terminology for stiffness here is the equilibrating axial force in the pipe when it is under unit pressure and the axial displacement at the boundary is put to zero. We had to separate this part from the actual axial stiffness with zero pressure, so that the value of internal pressure could vary as needed.

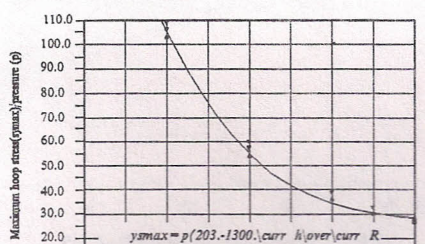

Fig.3 Maximum hoop stress curve valid for c/R=0 to .2

Figure 4 shows the results of the axial equilibrating force in non- dimensional form. For each ratio of L/R the relationship is idealized as a straight line with the change of c/R. Then the slopes of these lines are put to form a second order equation. As a result an equation for the axial force (stiffness) is deduced which is valid for the given

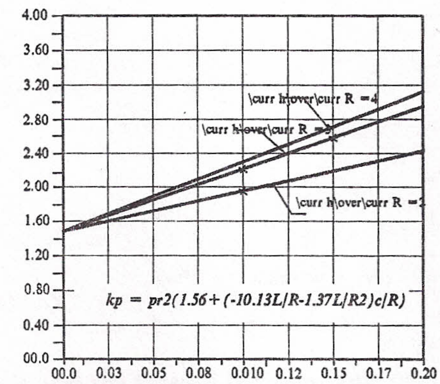

fig 4 : Axial stiffness for differeness for diferent values of c?R and L/R

ranges of the geometrical parameters.

$$Kp = p\,R^2\,\{ 1.56 + (-10.64 + 10.13\,L/R - 1.37\,L^2/R^2)\,c\} \quad (2)$$

b) Axial Stiffness of Bellows with no Internal Pressure

The parametric cases which were solved for unit internal pressure are solved here for a unit axial displacement and zeor pressure. It is noted that for all cases of $c/R > 0$ the meridional bending stresses are important when compared with the meridional membrane stresses. The axial stiffness is calculated as the equilibrating axial force due to a unit axial displacement.

Figure 5 shows the axial stiffness approximated as straight lines for different rations of L/R, and an equation is obtained which relates the axial stiffness for zero pressure to the material and geometric variables.

$$K = \frac{ERh}{LT}\,\{ -12.7 + (121.0 - 48.9\,L/R + 5.4\,L^2/R^2)\,c/R \}$$

where L_T is the total length of the bellows.

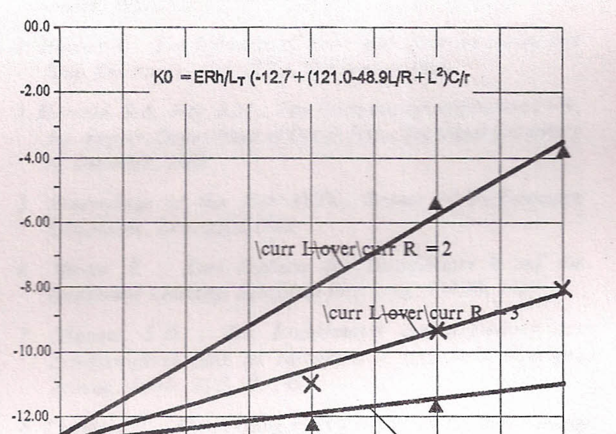

Fig 5 Axial stiffness for different values of c/R and L/R

4. BENDING STIFFNESS.

To find the external coupling moment on a pipe due to a unit rotation when solved as a shell, one has to go through two stages. Firstly, applying an end axial displacement of first harmonic type ($u_z = \cos\theta$) with zero rotation; and secondly, a unit moment also of first harmonic ($M_\phi = \cos\theta$) with zero axial displacement. Summing these two effects so that the lower and upper edges will correspond to an external relation, then summing the effects of the two cases N_o and M_ϕ to give the corresponding external moment which produces a unit rotation.

The results obtained in article (3) are made use of in reducing the number of parametric cases being tested for the bending stiffness. Correspondingly only one value ($h/R = .02$) is used, and the other parameters are given as:

c/R = 0.0, 0.1 and 0.2

L/R = 2.0, 3.2 and 4.0

Figure 6 shows the bending stiffness of corrugated pipes put into a linear form for different values of (L/R) with respect to changing given values of c, and given in an equation form as:

$$K_b = \frac{EhR^3}{L_T}\{19.5 + (-135.3 + 31.7\,L/R - 3.3\,L^2/R^2)\,c/R\} \quad (4)$$

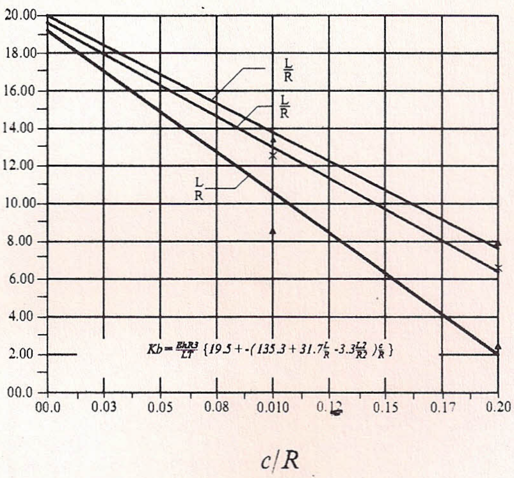

Fig 6 : Bending stiffness different values of C?r and L/R

5. PIPE EXPANSION PROBLEM

A test problem of a pipe line with an expansion joint is solved. Figure 7 shows the pipeline layout as well as its geometric and material properties. The curves given in figures 3 to 6 and their corresponding equations (1) to (4) are tested by solving the problem of the expansion joint (Bellows) separately. The obtained values for the maximum hoop stresses and axial stiffnesses were very close to the computer results, and either approach is acceptable within a reasonable error range.

The pipe and the bellows interaction is represented schematically by forces or stiffnesses of both the pipe and the bellows as shown in figure 8.

Therefore,

$P_P = -K_{pP} + K_{TP} + K_{oP}\,u$

$P_B = -K_{pB} + K_{TB} - K_{oB}\,u$

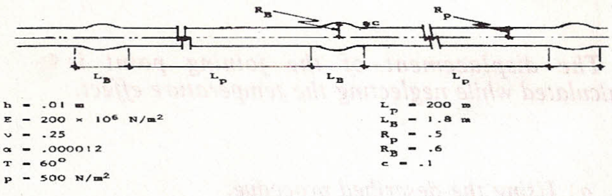

FIG. 7 PIPE LINE AND PIPE EXPANSION JOINT CONFIGURATION

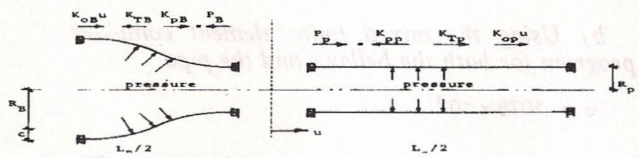

Fig 8 : Equilibrium of forces acting on the nodal circle connecting a pipe and an expansion joint (Bellows)

where:

P_P - Axial force on the pipe.

P_B - Axial force on the bellows.

K_{pP} - Axial force on the pipe due to pressure if ends are kept in position.

K_{oP} - Axial stiffness of the pipe with zero pressure.

K_{TP} - Axial force in the pipe due to temperature.

K_{pB} - Axial stiffness of the bellows with zero pressure.

K_{oB} - Axial stiffness of the bellows with zero pressure

K_{TB} - Axial force in the bellows due to temperature.

u - The displacement of the joining point of the pipe and the bellows, and its positive direction is as shown in figure 8.

In the equilibrium position the two forces P_P and P_B have to be equal. Then the following equation is obtained.

$$(K_{oB} + K_{oP})u = (K_{pP} - K_{pB}) + (K_{TB} - K_{TP})$$

The values of K_{oB}, K_{oP}, K_{pP} and K_{pB} are obtainable either by using the presented curves or their corresponding equations. The values of K_{TB} and K_{TP} are obtained by using the following equations:

$K_{TB} = K_{oB}\,\alpha T\,L_B = K_{oB}\,u_{TB}$

$K_{TP} = K_{oP}\,\alpha T\,L_P = K_{oP}\,u_{TP}$

For the test problem given in figure 7, these equilibrating forces are obtained for both bellows and the pipe.

Pipe	*Bellows.*
$K_{pP} = 195.$	$K_{pB} = 126.41$
$K_{oP} = .6345 \times 10^5$	$K_{oB} = 59.467 \times 10^5$
$K_{TP} = 4568.4$	$K_{TB} = 3854.46$

The displacement of the joining point is calculated while neglecting the temperature effect.

a) Using the described procedue.

$$u = 0.114 \times 10^{-4}$$

b) Using the mixed finite element computer program for both the bellows and the pipe.

$$u = .1075 \times 10^{-4}$$

The two results are reasonably close. The temperature effect is then taken into consideration, and the computed displacement is given as :

$$u = -0.1075 \times 10^{-3}$$

Accordingly the axial force and the maximum hoop stresses can be obtained in both the bellows and the pipe. Depending on the level of stresses and the flexibility requirements, the bellows is either accepted or another bellows with different dimensions is tried, until an optimum shape with enough flexibility and required stresses is reached.

6. CONCLUSION

In this paper a parametric study of pressurized pipe expansion joints (Bellows) of sinusoidal meridional shape is presented. Curves and equations governing the axial and bending stiffnesses of bellows and corrugated pipes are given. The presented curves, equations and numerical problem are hoped to be usefull to designers at least in the preliminary stage level.

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